**SUBJECT NAME** : **Discrete Mathematics**

**SUBJECT CODE** : **MA 2265**

**MATERIAL NAME** : **Formula Material**

**MATERIAL CODE** : **JM08ADM009**

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**Name of the Student:** **Branch:**



# Unit – I (Logic and Proofs)

1) **Truth Table**:

Conjunction

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

Disjunction

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

Conditional

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

Biconditional

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

Negation

| $p$ | $\sim p$ |
|---|---|
| T | F |
| F | T |

2) **Tautology and Contradiction**:

A Compound proposition $P = (P_1, P_2, \ldots P_n)$ where $P_1, P_2, \ldots P_n$ variables are called tautology if it is true for every truth assignment for $P_1, P_2, \ldots P_n$.

$P$ is called a Contradiction if it is false for every truth assignment for $P_1, P_2, \ldots P_n$.

If a proposition is neither a tautology nor a Contradiction is called contingency.

3) **Laws of algebra of proposition**:

| **Name of Law** | **Primal form** | **Dual form** |
|---|---|---|

| | | |
|---|---|---|
| Idempotent law | $p \vee p \equiv p$ | $p \wedge p \equiv p$ |
| Identity law | $p \vee F \equiv p$ | $p \wedge T \equiv p$ |
| Dominant law | $p \vee T \equiv T$ | $p \wedge F \equiv F$ |
| Complement law | $p \vee \sim p \equiv T$ | $p \wedge \sim p \equiv F$ |
| Commutative law | $p \vee q \equiv q \vee p$ | $p \wedge q \equiv q \wedge p$ |
| Associative law | $(p \vee q) \vee r \equiv p \vee (q \vee r)$ | $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ |
| Distributive law | $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ | $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ |
| Absorption law | $p \vee (p \wedge q) \equiv p$ | $p \wedge (p \vee q) \equiv p$ |
| Demorgan's law | $\sim (p \vee q) \equiv \sim p \wedge \sim q$ | $\sim (p \wedge q) \equiv \sim q \vee \sim p$ |
| Double Negation law | $\sim\sim p \equiv p$ | |

4) **Equivalence involving Conditionals**:

| Sl.No. | Propositions |
|---|---|
| 1. | $p \rightarrow q \equiv \sim p \vee q$ |
| 2. | $p \rightarrow q \equiv \sim q \rightarrow \sim p$ |
| 3. | $p \vee q \equiv \sim p \rightarrow q$ |
| 4. | $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$ |
| 5. | $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$ |

5) **Equivalence involving Biconditionals**:

| Sl.No. | Propositions |
|---|---|
| 1. | $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ |
| 2. | $p \leftrightarrow q \equiv \sim p \leftrightarrow \sim q$ |

| | |
|---|---|
| 3. | $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$ |
| 4. | $\sim (p \leftrightarrow q) \equiv p \leftrightarrow \sim q$ |

6) **Tautological Implication**:
$A \Rightarrow B$ if and only if $A \rightarrow B$ is tautology. (i.e) To prove $A \Rightarrow B$, it enough to prove $A \rightarrow B$ is tautology.

7) **The Theory of Inferences**:
The analysis of the validity of the formula from the given set of premises by using derivation is called "theory of inferences"

8) **Rules for inferences theory**:
**Rule P**:
A given premise may be introduced at any stage in the derivation.
**Rule T**:
A formula S may be introduced in a derivation if S is tautologically implied by one or more of the preceding formulae in the derivation.
**Rule CP**:
If we can drive S from R and a set of given premises, then we can derive $R \rightarrow S$ from the set of premises alone. In such a case R is taken as an additional premise (assumed premise). Rule CP is also called the deduction theorem.

9) **Indirect Method of Derivation**:
Whenever the assumed premise is used in the derivation, then the method of derivation is called indirect method of derivation.

10) **Table of Logical Implications:**

| Name of Law | Primal form |
|---|---|
| Simplification | $p \wedge q \Rightarrow p$<br>$p \wedge q \Rightarrow q$ |
| Addition | $p \Rightarrow p \vee q$<br>$q \Rightarrow p \vee q$ |
| Disjunctive Syllogism | $\sim p \wedge (p \vee q) \Rightarrow q$<br>$\sim q \wedge (p \vee q) \Rightarrow p$ |
| Modus Ponens | $p \wedge (p \rightarrow q) \Rightarrow q$ |
| Modus Tollens | $(p \rightarrow q) \wedge \sim q \Rightarrow \sim p$ |
| Hypothetical Syllogism | $(p \rightarrow q) \wedge (q \rightarrow r) \Rightarrow p \rightarrow r$ |

$$p \to q \Rightarrow \ \sim q \to \sim p$$

# Unit – II (Combinatorics)

1) **Principle of Mathematical Induction**:
   Let $P(n)$ be a statement or proposition involving for all positive integers n.
   **Step 1**: $P(1)$ is true.
   **Step2**: Assume that $P(k)$ is true.
   **Step3**: We have to prove $P(k+1)$ is true.
2) **Principle of Strong induction**.
   Let $P(n)$ be a statement or proposition involving for all positive integers n.
   **Step 1**: $P(1)$ is true.
   **Step2**: Assume that $P(n)$ is true for all integers $1 \le n \le k$.
   **Step3**: We have to prove $P(k+1)$ is true.
3) **The Pigeonhole Principle**:
   If $n$ pigeons are assigned to $m$ pigeonholes and $m < n$, than at least one pigeonhole contains two or more pigeons.
4) **The Extended Pigeonhole Principle**:
   If $n$ pigeons are assigned to $m$ pigeonholes than one pigeonhole must contains at least $\left[\frac{(n-1)}{m}\right]+1$ pigeons.
5) **Recurrence relation**:
   An equation that expresses $a_n$, the general term of the sequence $\{a_n\}$ in terms of one or more of the previous terms of the sequence, namely $a_0, a_1, \ldots a_{n-1}$, for all integers $n$ is called a recurrence relation for $\{a_n\}$ or a difference equation.
6) **Working rule for solving homogeneous recurrence relation**:
   **Step 1**: The given recurrence relation of the form
   $$C_0(n)a_n + C_1(n)a_{n-1} + \ldots + C_k(n)a_{n-k} = 0$$
   **Step 2**: Write the characteristic equation of the recurrence relation
   $$C_0 r_{n+k} + C_1 r_{n+(k+1)} + \ldots + C_k r_n = 0$$
   **Step 3**: Find all the roots of the characteristic equation namely $r_1, r_2, \ldots r_k$.
   **Step 4**:
   Case (i): If all the roots are distinct then the general solution is
   $$a_n = b_1 r_1^n + b_2 r_2^n + \ldots + b_k r_k^n$$
   Case (ii): If all the roots are equal then the general solution is
   $$a_n = \left(b_1 + nb_2 + n^2 b_3 + \ldots\right) r^n$$

# Unit – III (Graph Theory)

1) **Graph**:

   A graph G=(V,E) consists of two sets $V=\{v_1,v_2,...v_\upsilon\}$ , called the set of vertices and $E=\{e_1,e_2,...e_\in\}$ , called the set of edges of G.

2) **Simple graph**:

   A graph is said to be simple graph if it has no loops and parallel edges. Otherwise it is multi graph.

3) **Regular graph**:

   If every vertex of a simple graph has the same degree, then the graph is called a regular graph. If every vertex in a regular graph has degree n, then the graph is called n-regular.

4) **Complete graph**:

   A simple graph in which each pair of distinct vertices is joined by an edge is called a complete graph. The complete graph on "n" vertices is denoted by $K_n$ .

5) **Pendent vertex and Pendent edge**:

   A vertex with degree one is called a pendent vertex and the only edge which is incident with a pendent vertex is called the pendent edge.

6) **Matrix representation of a graph**:

   There are two ways of representing a graph by a matrix namely adjacent matrix and incidence matrix as follows:

   **Adjacency matrices**:

   Let G be a graph with n vertices, then the adjacency matrix, $A_G=(A_{ij})$ defined by $A_{ij}=\begin{cases}1, & \text{if } u_i,\ v_j \text{ are adjacent}\\ 0, & \text{otherwise}\end{cases}$.

   **Incidence matrix**:

   Let G be a graph with n vertices, then the incidence matrix of G is an n x e matrix $B_G=(B_{ij})$ defined by $B_{ij}=\begin{cases}1, & \text{if } j^{th} \text{ edge is incident on the } i^{th} \text{vert ex}\\ 0, & \text{otherwise}\end{cases}$

7) **Bipartite graph**:

   A graph G=(V,E) is called a bipartite graph if its vertex set V can be partitioned into two subsets $V_1$ and $V_2$ such that each edge of G connects and vertex of $V_1$ to a vertex of $V_2$ . In other words, no edge joining two vertices, in $V_1$ or two vertices in $V_2$ .

8) **Isomorphism of a graph**:

   The simple graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ are isomorphic if there is a one to one and onto function $f$ from $V_1$ to $V_2$ with the property that a and b are adjacent in $G_1$ if and only if $f(a)$ and $f(b)$ are adjacent in $G_2$ , for all a and b in $V_1$ .

9) **Complementary and Self complementary graph**:

Let $G$ be a graph. The complement $\overline{G}$ of $G$ is defined by any two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in $G$ .

$G$ is said to be a self complementary graph if $G$ is isomorphic to $\overline{G}$ .

10) **Connected graph**:

A graph G is said to be connected if there is at least one path between every pair of vertices in G. Otherwise G is disconnected. A disconnected graph consists of two or more connected sub graphs and each of them is called a component. It is denoted by $\omega(G)$ .

11) **Strongly Connected and Weakly Connected graph:**

12) **Cut edge**:

A cut edge of a graph G is an edge "e" such that $\omega(G-e) > \omega(G)$ . (i.e) If G is connected and "e" is a cut edge of G, then $G-e$ is disconnected.

13) **Cut vertex**:

A cut vertex of a graph G is a vertex "v" such $\omega(G-v) > \omega(G)$ . (i.e) If G is connected and "v" is a cut vertex of G, then $G-v$ is disconnected.

14) **Define vertex connectivity**.

The connectivity $\kappa(G)$ of G is the minimum "k" for which G has a k-vertex cut. If G is either trivial or disconnected then $\kappa(G)=0$ .

15) **Define edge connectivity**.

The edge connectivity $\kappa'(G)$ of G is the minimum "k" for which G has a k-edge cut. If G is either trivial or disconnected then $\kappa'(G)=0$

16) **Define Eulerian graph**.

A path of graph G is called an Eulerian path, if it includes each edge of G exactly once. A circuit of a graph G is called an Eulerian circuit, if it includes each edge of G exactly one. A graph containing an Eulerian circuit is called an Eulerian graph.

17) **Define Hamiltonian graph**.

A simple path in a graph G that passes through every vertex exactly once is called a Hamilton path. A circuit in a graph G that passes through every vertex exactly once is called a Hamilton circuit. A graph containing a Hamiltonian circuit.

# Unit – IV (Algebraic Structures)

1) **Semi group**:

If $G$ is a non-empty set and * be a binary operation on G, then the algebraic system $(G,*)$ is called a semi group, if G is closed under * and * is associative.

**Example**: If $Z'$ is the set of positive even numbers, then $(Z',+)$ and $(Z',\times)$ are semi groups.

2) **Monoid**:

If a semi group $(G,*)$ has an identity element with respect to the operation *, then $(G,*)$ is called a monoid. It is denoted by $(G,*,e)$.

**Example**: If N is the set of natural numbers, then $(N,+)$ and $(N,\times)$ are monoids with the identity elements 0 and 1 respectively. $(Z',+)$ and $(Z',\times)$ are semi groups with out monoids, where $Z'$ is the set of all positive even numbers

3) **Sub semi groups**:

If $(G,*)$ is a semi group and $H \subseteq G$ is called under the operation *, then $(H,*)$ is called a sub semi group of $(G,*)$.

**Example**: If the set $E$ of all even non-negative integers, the $(E,+)$ is a sub semi group of the semi group $(N,+)$, where $N$ is the set of natural numbers.

4) **Semi group homomorphism**:

If $(G,*)$ and $(G',\Delta)$ are two semi groups, then a mapping $f : G \to G'$ is called a semi group homomorphism, if for any $a,b \in G$, $f(a*b) = f(a)\Delta f(b)$. A homomorphism f is called isomorphism if f is 1-1 and onto.

5) **Group**:

If $G$ is a non-empty set and * is a binary operation of $G$, then the algebraic system $(G,*)$ is called a group if the following conditions are satisfied.

(i) Closure property
(ii) Associative property
(iii) Existence of identity element
(iv) Existence of inverse element

Example: $(Z,+)$ is a group and $(Z,\bullet)$ is not a group.

6) **Abelian group**:

A group $(G,*)$, in which the binary operation * is commutative, is called a commutative group or abelian group.

**Example**: The set of rational numbers excluding zero is an abelian group under the multiplication.

7) **Coset**:

If H is a subgroup of a group G under the operation *, then the set aH, where $a \in G$, define by $aH = \{a*h / h \in H\}$ is called the left coset of H in G generated by the element $a \in G$. Similarly the set Ha is called the right coset of H in G generated by the element $a \in G$.

**Example**: $G=\{1,-1,i,-i\}$ be a group under multiplication and $H=\{1,-1\}$ is a subgroup of G. The right cosets are $1H=\{1,-1\}$, $-1H=\{-1,1\}$, $iH=\{i,-i\}$ and $-iH=\{-i,i\}$.

8) **Lagrange's theorem**:
The order of each subgroups of a finite group is a divisor of a order of a group.

9) **Cyclic group**:
A group $(G,*)$ is said to be cyclic, if $\exists$ and element $a\in G$ such that every element of G generated by a. (i.e) $G=<a>=\{1,a,a^2,...a^n=e\}$.
**Example**: $G=\{1,-1,i,-i\}$ is a cyclic group under the multiplication. The generator is $i$, because $i^4=1,\ i^2=-1,\ i,\ i^3=-i$.

10) **Normal subgroup**:
A subgroup H of the group G is said to be normal subgroup under the operation *, if for any $a\in G$, $aH=Ha$.

11) **Kernel of a homomorphism**:
If f is a group homomorphism from $(G,*)$ and $(G',\Delta)$, then the set of element of $G$, which are mapped into $e'$, the identity element of $G'$, is called the kernel of the homomorphism f and denoted by $\ker(f)$.

12) **Fundamental theorem of homomorphism**:
If f is a homomorphism of $G$ on to $G'$ with kernel K, then $G/K$ is isomorphic to $G'$.

13) **Cayley's theorem**:
Every finite group of order $n$ is isomorphic to a permutation group of degree $n$.

14) **Ring**:
An algebraic system $(S,+,\bullet)$ is called a ring if the binary operations $+$ and $\bullet$ on $S$ satisfy the following properties.
   (i) $(S,+)$ is an abelian group
   (ii) $(S,\bullet)$ is a semi group
   (iii) The operation $\bullet$ is distributive over $+$.

**Example**: The set of all integers $Z$, and the set of all rational numbers $R$ are rings under the usual addition and usual multiplication.

15) **Commutative ring:**

16) **Integral domain**:
A commutative ring without zero divisor is called Integral domain.
**Example**: (i) $(R,+,\bullet)$ is an integral domain, since $a,b\in R$ such that $a\neq 0,\ b\neq 0$ then $ab\neq 0$. (ii) $(Z_{10},+_{10},\times_{10})$ is not an integral domain,

because $2,3 \in Z_{10}$ and $2 \times_{10} 5 = 0$. Therefore 2 and 5 are zero divisors.

17) **Field**:

A commutative reing $(S,+,\bullet)$ which has more than one element such that every non-zero element of $S$ has a multiplicative inverse in $S$ is called a field.

**Example**: The ring of rational numbers $(Q,+,\bullet)$ is a field since it is a commutative ring and each non-zero element is inversible.

# Unit – V (Lattices and Boolean algebra)

1) **Partially ordered set (Poset)**:

A relation R on a set A is called a partial order relation, if R is reflexive, antisymmetric and transitive. The set A together with partial order relation R is called partially ordered set or poset.

**Example**: The greater than or equal to $(\geq)$ relation is a partial ordering on the set of integers $Z$.

2) **Lattice**:

A lattice is a partially ordered set $(L,\leq)$ in which every pair of elements $a,b \in L$ has a glb and lub.

3) **Sub-lattice**:

4) **General formula**:

i) $\text{glb}\{a,b\} = a * b = a \wedge b$

ii) $\text{lub}\{a,b\} = a \oplus b = a \vee b$

iii) $a * b \leq a$ & $a * b \leq b$ $\qquad$ $a \oplus b \geq a$ & $a \oplus b \geq b$

iv) If $a \leq b \Rightarrow a * b = a$, $a \oplus b = b$

5) **Properties**:

| Name of Law | Primal form | Dual form |
|---|---|---|
| Idempotent law | $a * a = a$ | $a \oplus a = a$ |
| Commutative law | $a * b = b * a$ | $a \oplus b = b \oplus a$ |
| Associative law | $(a * b) * c = a * (b * c)$ | $(a \oplus b) \oplus c = a \oplus (b \oplus c)$ |

| | | |
|---|---|---|
| Distributive law | $p*(q\oplus r)\equiv(p*q)\oplus(p*r)$ | $p\oplus(q*r)\equiv(p\oplus q)*(p\oplus r)$ |
| Absorption law | $a*(a\oplus b)=a$ | $a\oplus(a*b)=a$ |
| Complement | $a*a'=0$ | $a\oplus a'=1$ |
| Demorgan's law | $(a*b)'=a'\oplus b'$ | $(a\oplus b)'=a'*b'$ |
| Double Negation law | $\sim\sim p\equiv p$ | |

6) **Complemented Lattices**:
A Lattice $(L,*,\oplus)$ is said to be complemented if for any $a\in L$, there exist $a'\in L$, such that $a*a'=0$ and $a\oplus a'=1$.

7) **Demorgan's laws**:
Let $(L,*,\oplus)$ be the complemented lattice, then $(a*b)'=a'\oplus b'$ and $(a\oplus b)'=a'*b'$.

8) **Complete Lattice**:
A lattice $(L,*,\oplus)$ is complete if for all non-empty subsets of L, there exists a glb and lub.

9) **Lattice Homomorphism**:
Let $(L,*,\oplus)$ and $(S,\wedge,\vee)$ be two lattices. A mapping $g:L\to S$ is called lattices homomorphism if $g(a*b)=g(a)\wedge g(b)$ and $g(a\oplus b)=g(a)\vee g(b)$.

10) **Modular Lattice**:
A lattice $(L,*,\oplus)$ is said to be modular if for any $a,b,c\in L$

i) $a\le c \Rightarrow a\oplus(b*c)=(a\oplus b)*c$

ii) $a\ge c \Rightarrow a*(b\oplus c)=(a*b)\oplus c$

11) **Chain in Lattice**:
Let $(L,\le)$ be a Chain if

i) $a\le b$ or $a\le c$ and

ii) $a\ge b$ and $a\ge c$

12) **Condition for the algebraic lattice**:
A lattice $(L,*,\oplus)$ is said to be algebraic if it satisfies Commutative Law, Associative Law, Absorption Law and Existence of Idempotent element.

13) **Isotone property**:

Let $(L,*,\oplus)$ be a lattice. The binary operations $*$ and $\oplus$ are said to possess isotone property if $b \le c \Rightarrow \begin{array}{l} a*b \le a*c \\ a\oplus b \le a\oplus c \end{array}$.

14) **Boolean Algebra**:
A Boolean algebra is a lattice which is both complemented and distributive. It is denoted by $(B,*,\oplus)$.

---- *All the Best* ----