NAME OF THE SUBJECT : **Mathematics**

NAME OF THE MATERIAL : **Basic Formulas**

MATERIAL CODE : **HG13AUM101**

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Name of the Student: Branch:

Algebra

1. **Binomial Expansion:**

$$\frac{1}{1-x} = 1+x+x^2+x^3+\ldots$$

$$\frac{1}{1+x} = 1-x+x^2-x^3+\ldots$$

$$\frac{1}{(1+x)^2} = 1-2x+3x^2-4x^3+\ldots$$

$$\frac{1}{(1-x)^2} = 1+2x+3x^2+4x^3+\ldots$$

The above formulas are valid if $|x|<1$.

2. **General:**

$$(a+b)^2 = a^2+b^2+2ab$$

$$(a-b)^2 = a^2+b^2-2ab$$

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$

$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$$

$$(a-b)^3 = a^3-3a^2b+3ab^2-b^3$$

$$(a+b)(a-b) = a^2-b^2$$

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$

$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

$$(x+a)(x+b) = x^2+(a+b)x+ab$$

$$(x+a)(x+b)(x+c) = x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$$

3. **Converting to Perfect Square:**

$$x^2+ax = \left(x+\frac{a}{2}\right)^2-\frac{a^2}{4}$$

$$x^2-ax = \left(x-\frac{a}{2}\right)^2-\frac{a^2}{4}$$

Trigonometry

1. **Exponential Form:**

$$e^{i\theta} = \cos\theta + i\sin\theta$$
$$e^{i\theta} = \cos\theta + i\sin\theta$$
$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$$
$$\sin\theta = \frac{e^{i\theta}+e^{-i\theta}}{2i}$$

2. **Inter Relation Formulae:**

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta},\quad \cos\theta = \frac{1}{\sec\theta},\quad \tan\theta = \frac{1}{\cot\theta},\quad \tan\theta = \frac{\sin\theta}{\cos\theta}.$$

3. **Identities:**

$$\sin^2\theta+\cos^2\theta = 1$$
$$\sec^2\theta = 1+\tan^2\theta$$
$$\operatorname{cosec}^2\theta = 1+\cot^2\theta$$

4. **Compound Angel Formulae:**

$$\sin(A+B) = \sin A\cos B+\cos A\sin B$$
$$\sin(A-B) = \sin A\cos B-\cos A\sin B$$
$$\cos(A+B) = \cos A\cos B-\sin A\sin B$$
$$\cos(A-B) = \cos A\cos B+\sin A\sin B$$
$$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$$
$$\tan(A-B) = \frac{\tan A-\tan B}{1+\tan A\tan B}$$

5. **Product Formulae:**

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A+B)-\sin(A-B)\right]$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A+B)+\cos(A-B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]$$

6. **Special Formulae:**

$$\sin 2A = 2\sin A\cos A \qquad \sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1+\tan^2 A}$$

$$1+\cos 2A = 2\cos^2 A$$

$$1-\cos 2A = 2\sin^2 A$$

$$\sin 3A = 3\sin A - 4\sin^3 A$$

$$\cos 3A = 4\cos^3 A - 3\cos A$$

7. **Power to Non Power Formulae:**

$$\cos^2\theta = \frac{1}{2}\left(1+\cos 2\theta\right) \qquad 1+\cos\theta = 2\cos^2\frac{\theta}{2}$$

$$\sin^2\theta = \frac{1}{2}\left(1-\cos 2\theta\right) \qquad 1-\cos\theta = 2\sin^2\frac{\theta}{2}$$

$$\cos^3\theta = \frac{1}{4}\left(3\cos\theta+\cos 3\theta\right)$$

$$\sin^3\theta = \frac{1}{4}\left(3\sin\theta-\sin 3\theta\right)$$

8. **Sum to Product Formulae:**

$$\sin C+\sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\sin C-\sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C+\cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos C-\cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

9. **Hyperbolic Functions**

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh^2 \theta - \sinh^2 \theta = 1$$

Logarithmic

1. **Basic Formulae**

$$\log(mn) = \log m + \log n$$

$$\log\left(\frac{m}{n}\right) = \log m - \log n$$

$$\log\left(\frac{1}{m}\right) = -\log m$$

$$\log 1 = 0$$

Analytical Geometry

1. Equation of Straight Lines:

   General Form:

$$ax + by + c = 0$$

   Intersect Form:

$$\frac{x}{a} + \frac{y}{b} = 1$$

   Slope and Intersect Form:

$$y = mx + c$$

2. Perpendicular Distance:

   Perpendicular Distance from a point (x_1, y_1) to the straight line $ax + by + c = 0$

$$= \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

Perpendicular Distance from a point (x_1, y_1, z_1) to the plane $ax+by+cz+d=0$

$$= \left| \frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}} \right|$$

3. Equation of Circle:

   General Form:

   $$x^2+y^2+2gx+2fy+d=0$$

   Centre $(-g,-f)$ and Radius $= \sqrt{g^2+f^2-d}$

   Circle with Centre Origin and Radius a:

   $$x^2+y^2=a^2$$

   Circle with Centre Non Origin and Radius a:

   $$(x-h)^2+(y-k)^2=a^2, \quad \text{Centre } (h,k)$$

4. Equation of Sphere:

   General Form:

   $$x^2+y^2+z^2+2ax+2by++2cz+d=0$$

   Centre $(-a,-b,-c)$ and Radius $= \sqrt{a^2+b^2+c^2-d}$

   Circle with Centre Origin and Radius a:

   $$x^2+y^2+z^2=a^2$$

   Circle with Centre Non Origin and Radius a:

   $$(x-l)^2+(y-m)^2+(z-n)^2=a^2, \quad \text{Centre } (l,m,n)$$

5. Equation of Curve:

Sl.No.	Curve	Cartesian Equation	Parametric Equation
1	Parabola	$y^2=4ax$	$x=at^2$ $y=2at$
		$x^2=4ay$	$x=2at$ $y=at^2$
2	Ellipse	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$x=a\cos\theta$ $y=b\sin\theta$

3	Hyperbola	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$x=a\sec\theta$ $y=b\tan\theta$
4	Rectangular Hyperbola	$xy=c^2$	$x=ct$ $y=\frac{c}{t}$
5	Astroid	$x^{2/3}+y^{2/3}=a^{2/3}$	$x=a\cos^3\theta$ $y=a\sin^3\theta$

Differential Calculus

1. Derivative Formula:

Sl.No.	y	$\frac{dy}{dx}$
1.	Constant	0
2.	x^n	nx^{n-1}
3.	x	1
4.	$\frac{1}{x^n}$ $\frac{1}{x}$	$\frac{-n}{x^{n+1}}$ $\frac{-1}{x^2}$
5.	$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
6.	$e^{(ax+b)}$ e^x a^x	$ae^{(ax+b)}$ e^x $a^x\log a$
7.	$\log(ax+b)$ $\log x$ $\log_{10} x$	$\frac{a}{ax+b}$ $\frac{1}{x}$ $\frac{1}{x}\log_{10} e$

8.	$\sin(ax+b)$ $\sin x$	$a\cos(ax+b)$ $\cos x$
9.	$\cos(ax+b)$ $\cos x$	$-a\sin(ax+b)$ $-\sin x$
10.	$\tan(ax+b)$ $\tan x$	$a\sec^2(ax+b)$ $\sec^2 x$
11.	$\operatorname{cosec}(ax+b)$ $\operatorname{cosec} x$	$-a\operatorname{cosec}(ax+b)\cot(ax+b)$ $-\operatorname{cosec} x\cot x$
12.	$\sec(ax+b)$ $\sec x$	$a\sec(ax+b)\tan(ax+b)$ $\sec x\tan x$
13.	$\cot(ax+b)$ $\cot x$	$-a\operatorname{cosec}^2(ax+b)$ $-\operatorname{cosec}^2 x$
14.	$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
15.	$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$
16.	$\tan^{-1} x$	$\frac{1}{1+x^2}$

2. Special Formulae:

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}(uvw) = uv\frac{dw}{dx} + vw\frac{du}{dx} + wu\frac{dv}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Integral Calculus

1. $\int (ax+b)^n dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1}+c$ $\qquad \int x^n dx = \frac{x^{n+1}}{n+1}+c$
2. $\int e^{ax+b}dx = \frac{e^{ax+b}}{a}+c$ $\qquad \int e^x dx = e^x+c$
3. $\int \frac{1}{ax+b}dx = \frac{1}{a}\log(ax+b)+c$ $\qquad \int \frac{1}{x}dx = \frac{1}{x}+c$

   $\int \frac{1}{x^n}dx = \frac{-1}{(n-1)x^{n-1}}+c$
4. $\int \sqrt{x}dx = \frac{2x^{3/2}}{3}+c$
5. $\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b)+c$ $\qquad \int \sin x\,dx = -\cos x+c$
6. $\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b)+c$ $\qquad \int \cos x\,dx = \sin x+c$
7. $\int \sec x\,dx = \log(\sec x+\tan x)+c$
8. $\int \operatorname{cosec} x\,dx = \log(\operatorname{cosec}x+\cot x)+c$
9. $\int \tan(ax+b)dx = \frac{1}{a}\log\sec(ax+b)+c$ $\qquad \int \tan x\,dx = \log\sec x+c$
10. $\int \cot(ax+b)dx = \frac{1}{a}\log\sin(ax+b)+c$ $\qquad \int \cot x\,dx = \log\sin x+c$
11. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+c$ $\qquad \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x+c$
12. $\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right)+c$ $\quad (or)$ $\quad \log\left[x+\sqrt{x^2-a^2}\right]+c$
13. $\int \frac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1}\left(\frac{x}{a}\right)+c$ $\quad (or)$ $\quad \log\left[x+\sqrt{x^2+a^2}\right]+c$
14. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$ $\qquad \int \frac{dx}{1+x^2} = \tan^{-1}x+c$
15. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)+c$
16. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)+c$
17. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c$

18. $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right)+c$

(or)

$= \frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left[x+\sqrt{a^2+x^2}\right]+c$

19. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right)+c$

(or)

$= \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left[x+\sqrt{x^2-a^2}\right]+c$

20. $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right]$

21. $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx + b\sin bx\right]$

22. Reduction Formulae

$\int_0^{\frac{\pi}{2}} \cos^n x\,dx$ **(or)** $\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \frac{n-1}{n}\frac{n-3}{n-2}\frac{n-5}{n-4}\cdots\frac{2}{3}.1$ **[if n is odd]**

$= \frac{n-1}{n}\frac{n-3}{n-2}\frac{n-5}{n-4}\cdots\frac{1}{2}.\frac{\pi}{2}$ **[if n is even]**

$\int_0^{\frac{\pi}{2}} \sin^m x\cos^n x\,dx = \frac{\left[(m-1)(m-3)\ldots\right]\left[(n-1)(n-3)\ldots\right]}{(m+n)(m+n-2)(m+n-4)\ldots}$

$= \frac{\left[(m-1)(m-3)\ldots\right]\left[(n-1)(n-3)\ldots\right]}{(m+n)(m+n-2)(m+n-4)\ldots}$ **[Both m and n are even]**

23. $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ **[if $f(x)$ is an even function]**

$= 0$ **[if $f(x)$ is an odd function]**

24. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

25. $\int_a^b f(x)dx = -\int_b^a f(a-x)dx$

26. Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

27. Bernoulli's Formulae

$$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \ldots$$

28. Gamma Function:

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1}\,dx$$

$$\frac{\Gamma(n)}{a^n} = \int_0^\infty e^{-ax} x^{n-1}\,dx$$

$$\Gamma(n) = (n-1)!$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

Sequence and Series

1. $\underset{x\to a}{Lt}\left(\frac{x^n - a^n}{x-a}\right) = na^{n-1}$
2. $\underset{x\to 0}{Lt}\frac{\sin x}{x} = 1$
3. $\underset{x\to 0}{Lt}\left(1+\frac{x}{n}\right)^n = e^x$
4. $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$
5. $e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots$
6. $\sin x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots$
7. $\cos x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots$
8. $\log\frac{1}{1-x} = x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots$
9. $1 + a + a^2 + a^3 + \ldots + a^n = \frac{1-a^{n+1}}{1-a}$

----All the Best----