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Name of the Student: Branch:

UNIT – 1

1. Newton's method (0r) Newton-Raphson method: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
2. Fixed point iteration (or) Iterative formula (or) Simple iteration method
   $x_{n+1} = g(x_n)$, *where* $x = g(x)$
3. The rate of convergence in N-R method is of order 2.
4. Condition for convergence of N-R method is $|f(x)f''(x)| < |f'(x)|^2$
5. Condition for the convergence of iteration method is $|g'(x)| < 1$
6. Gauss elimination & Gauss-Jordon are direct methods and Gauss-Seidal and Gauss-Jacobi are iterative methods.
7. Power method, To find numerically largest eigen value $Y_{n+1} = AX_n$

UNIT – 2

1. Lagrange's interpolation formula is
   $$y = f(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)}y_1 + \ldots + \frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})}y_n$$
2. Inverse of Lagrange's interpolation formula is
   $$x = f(y) = \frac{(y-y_1)(y-y_2)\ldots(y-y_n)}{(y_0-y_1)(y_0-y_2)\ldots(y_0-y_n)}x_0 + \frac{(y-y_0)(y-y_2)\ldots(y-y_n)}{(y_1-y_0)(y_1-y_2)\ldots(y_1-y_n)}x_1 + \ldots + \frac{(y-y_0)(y-y_1)\ldots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)\ldots(y_n-y_{n-1})}x_n$$
3. Newton's divided difference interpolation formula is
   $$f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + \ldots$$

4. Newton's forward difference formula is
$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \dots$$
where $u = \frac{x - x_0}{h}$
5. Newton's backward difference formula is
$$y(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \dots$$
where $v = \frac{x - x_n}{h}$

6. Interpolation with a cubic spline
$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}[y_{i-1} - 2y_i + y_{i+1}]$$
$$S(x) = \frac{1}{6h}\left[(x_i - x)^3 M_{i-1} + (x - x_{i-1})^3 M_i\right] + \frac{1}{h}(x_i - x)\left[y_{i-1} - \frac{h^2}{6}M_{i-1}\right]$$
$$+ \frac{1}{h}(x - x_{i-1})\left[y_i - \frac{h^2}{6}M_i\right]$$
for $i = 1, 2, \dots (n-1)$ and $x_{i-1} \le x \le x_i$

UNIT – 3

1. Newton's forward formula to find the derivatives
$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2u-1}{2}\Delta^2 y_0 + \frac{3u^2 - 6u + 2}{6}\Delta^3 y_0 + \dots\right]$$
$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \frac{6u^2 - 18u + 11}{12}\Delta^4 y_0 + \dots\right]$$
$$\frac{d^3y}{dx^3} = \frac{1}{h^3}\left[\Delta^3 y_0 + \frac{12u - 18}{12}\Delta^4 y_0 + \dots\right]$$ where $u = \frac{x - x_0}{h}$
2. Newton's forward formula to find the derivatives at $x = x_0$
$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \dots\right]$$
$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 + \dots\right]$$
$$\left(\frac{d^3y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + \dots\right]$$

3. Newton's backward formula to find the derivatives
$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{2v+1}{2}\nabla^2 y_n + \frac{3v^2 + 6v + 2}{6}\nabla^3 y_n + \dots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + (v+1)\nabla^3 y_n + \frac{6v^2+18v+11}{12}\nabla^4 y_n + \dots\right]$$

$$\frac{d^3y}{dx^3} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{12v+18}{12}\nabla^4 y_n + \dots\right] \text{ where } v = \frac{x-x_n}{h}$$

4. Newton's backward formula to find the derivatives at $x = x_n$

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \dots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \dots\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_n} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{3}{2}\nabla^4 y_n + \dots\right]$$

5. Trapezoidal Rule

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \dots)\right]$$

6. Simpson's 1/3rd Rule

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \dots) + 2(y_2 + y_4 + y_6 + \dots)\right]$$

7. Simpson's 3/8th Rule

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{3h}{8}\left[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + y_7 + \dots) + 2(y_3 + y_6 + y_9 + \dots)\right]$$

8. Romberg's formula for I_1 & I_2 is $I = I_2 + \left(\frac{I_2 - I_1}{3}\right)$

9. Error is the Trapezoidal formula is of the order h^2 and in the Simpson formula is of the order h^4.

10. Two points Gaussian Quadrature formula

$$\int_{-1}^{1} f(x)dx = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

11. Three points Gaussian Quadrature formula

$$\int_{-1}^{1} f(x)dx = \frac{5}{9}\left[f\left(-\sqrt{\frac{3}{5}}\right) + f\left(\sqrt{\frac{3}{5}}\right)\right] + \frac{8}{9}f(0)$$

12. If the range is not $(-1, 1)$ then the idea to solve the Gaussian Quadrature problem is $x = \left(\frac{b-a}{2}\right)z + \left(\frac{b+a}{2}\right)$

UNIT – 4

1. Taylor Series method

$$y_1 = y(x_0+h) = y_0 + \frac{h}{1!}y_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \ldots$$

and

$$y_2 = y(x_1+h) = y_1 + \frac{h}{1!}y_1' + \frac{h^2}{2!}y_1'' + \frac{h^3}{3!}y_1''' + \ldots$$

2. Euler's method

If $\frac{dy}{dx} = f(x,y);\ y(x_0) = y_0$

then $y(x_0+h) = y_0 + hf(x_0,y_0)$

and $y(x_1+h) = y_1 + hf(x_1,y_1)$

3. Modified Euler's method

If $\frac{dy}{dx} = f(x,y);\ y(x_0) = y_0$

then $y(x_0+h) = y_0 + h\, f\left[x_0+\frac{h}{2}, y_0+\frac{h}{2}f(x_0,y_0)\right]$

And $y(x_1+h) = y_1 + h\, f\left[x_1+\frac{h}{2}, y_1+\frac{h}{2}f(x_1,y_1)\right]$

4. Runge-kutta method of fourth order

First formula:

$$k_1 = hf(x_0,y_0) \qquad k_3 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}\right)$$

$$k_2 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right) \qquad k_4 = hf\left(x_0+h, y_0+k_3\right)$$

and $\Delta y = \frac{1}{6}\left[k_1+2k_2+2k_3+k_4\right]$ & $y(x_0+h) = y_0 + \Delta y$

For the second formula, replace 0 by 1 in the above formula.

5. Milne's Predictor formula: $y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y_{n-2}' - y_{n-1}' + 2y_n'\right]$

6. Milne's Corrector formula: $y_{n+1} = y_{n-1} + \frac{h}{3}\left[y_{n-1}' + 4y_n' + y_{n+1}'\right]$

7. <u>Adams-Bashforth method</u>:

Adam's Predictor formula: $y_{n+1} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}'\right]$

Adam's Corrector formula: $y_{n+1} = y_n + \frac{h}{24}\left[9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right]$

UNIT – 5

1. Solving ordinary diff. Eqn. by finite diff. method

   The given eqn. $y''(x) + f(x)y'(x) + g(x)y(x) = \phi(x)$

   $y''_i(x) = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$ & $y'_i(x) = \frac{y_{i+1} - y_{i-1}}{2h}$

2. Solving one dimensional heat eqn. by Bender –Schmidt's method [Explicit method]

   The given equation $\frac{\partial^2 u}{\partial x^2} = a\frac{\partial u}{\partial t}$

   then $u_{i,j+1} = \lambda u_{i+1,j} + (1-2\lambda)u_{i,j} + \lambda u_{i-1,j}$, where $\lambda = \frac{k}{ah^2}$

   for $\lambda = \frac{1}{2}$ the above equation becomes,

   $u_{i,j+1} = \frac{1}{2}\left[u_{i+1,j} + u_{i-1,j}\right]$, where $k = \frac{a}{2}h^2$

3. Solving one dimensional heat eqn. by Crank-Nicolson's method [Implicit method]

   The given equation $\frac{\partial^2 u}{\partial x^2} = a\frac{\partial u}{\partial t}$

   then $\lambda\left(u_{i+1,j+1} + u_{i-1,j+1}\right) - 2(\lambda+1)u_{i,j+1} = 2(\lambda-1)u_{i,j} - \lambda(u_{i+1,j} + u_{i-1,j})$

   where $\lambda = \frac{k}{ah^2}$

   for $\lambda = 1$ the above equation becomes,

   $u_{i,j+1} = \frac{1}{4}\left[u_{i-1,j+1} + u_{i+1,j+1} + u_{i-1,j} + u_{i+1,j}\right]$, where $k = ah^2$

4. Solving one dimensional wave eqn.

   The given equation $\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}$

   then $u_{i,j+1} = 2\left(1-\lambda^2 a^2\right)u_{i,j} + \lambda^2 a^2(u_{i+1,j} + u_{i-1,j}) - u_{i,j-1}$,

   where $\lambda = \frac{k}{h}$

for $\lambda^2 a^2 = 1$, the above equation becomes,

$$u_{i,j+1} = u_{i+1,j} + u_{i-1,j} - u_{i,j-1}, \qquad \text{where } k = \frac{h}{a}$$

5. Solving two dimensional Laplace equation

The given eqn. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (or) $\nabla^2 u = 0$

The Standard five point formula: [SFPF]

$$u_{i,j} = \frac{1}{4}\left[u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1}\right]$$

Diagonal five point formula: [DFPF]

$$u_{i,j} = \frac{1}{4}\left[u_{i-1,j-1} + u_{i-1,j+1} + u_{i+1,j-1} + u_{i+1,j+1}\right]$$

Liebamann's iteration process:

$$u_{i,j}^{(n+1)} = \frac{1}{4}\left[u_{i-1,j}^{(n+1)} + u_{i+1,j}^{(n)} + u_{i,j-1}^{(n)} + u_{i,j+1}^{(n+1)}\right]$$

6. Solving two dimensional Poisson equation

The given eqn. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)$ (or) $\nabla^2 u = f(x,y)$

To solve the above equation, we use the following formula

$$u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} - 4u_{i,j} = h^2 f(ih, jh)$$

---- *All the Best* ----