**SUBJECT NAME : Discrete Mathematics**

**SUBJECT CODE : MA 2265**

**MATERIAL NAME : Part – A questions**

**MATERIAL CODE : JM08AM1013**

**REGULATION : R2008**

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## Name of the Student: Branch:

## Unit – I (Logic and Proofs)

1. Define Tautology with an example.
2. Define a rule of Universal specification.
3. Find the truth table for the statement $P \to Q$.
4. Construct a truth table for the compound proposition $(p \to q) \to (q \to p)$.
5. Using truth table show that $P \vee (P \wedge Q) \equiv P$.
6. Construct the truth table for the compound proposition $(p \to q) \leftrightarrow (\neg p \to \neg q)$.
7. Given $P = \{2,3,4,5,6\}$, state the truth value of the statement $(\exists x \in P)(x+3=10)$.
8. Show that the propositions $p \to q$ and $\neg p \vee q$ are logically equivalent.
9. Show that $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent.
10. Prove that $p,\ p \to q,\ q \to r \ \Rightarrow\ r$.
11. Without using truth table show that $P \to (Q \to P) \Rightarrow \neg P \to (P \to Q)$.
12. Show that $(P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$ is a tautology.
13. Using truth table, show that the proposition $p \vee \neg(p \wedge q)$ is a tautology.

14. Is $\left(\neg p \wedge \left(p \vee q\right)\right) \rightarrow q$ a tautology?

15. Write the negation of the statement $\left(\exists x\right)\left(\forall y\right) p\left(x, y\right)$.

16. What are the negations of the statements $\forall x\left(x^2 > x\right)$ and $\exists x\left(x^2 = 2\right)$ ?

17. Give an indirect proof of the theorem " If $3n+2$ is odd, then $n$ is odd".

18. When do you say that two compound propositions are equivalent?

19. What are the contra positive, the converse and the inverse of the conditional statement "*If you work hard then you will be rewarded*".

20. Symbolically express the following statement.

    " It is not true that 5 star rating always means good food and good service ".

# Unit – II (Combinatorics)

1. State the principle of strong induction.

2. Use mathematical induction to show that $n! \geq 2^{n+1},\ n = 1, 2, 3, \ldots$.

3. Use mathematical induction to show that $1+2+3+\ldots+n = \dfrac{n(n+1)}{2}$.

4. If seven colours are used to paint 50 bicycles, then show that at least 8 bicycles will be the same colour.

5. Write the generating function for the sequence $1, a, a^2, a^3, a^4, \ldots$.

6. Find the recurrence relation for the Fibonacci sequence.

7. Find the recurrence relation satisfying the equation $y_n = A(3)^n + B(-4)^n$.

8. Solve the recurrence relation $y\left(k\right) - 8y\left(k-1\right) + 16y\left(k-2\right) = 0$ for $k \geq 2$, where $y(2) = 16$ and $y(3) = 80$.

9. Solve: $a_k = 3a_{k-1}$, for $k \geq 1$, with $a_0 = 2$.

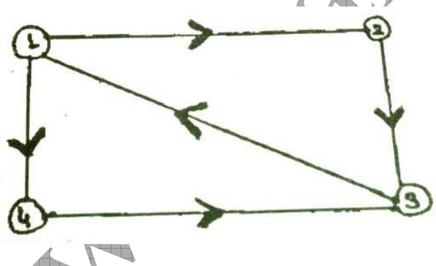
10. Find the number of non-negative integer solutions of the equation $x_1 + x_2 + x_3 = 11$.

11. State Pigeonhole principle.

12. What is well ordering principle?
13. In how many ways can all the letters in MATHEMATICAL be arranged.
14. What is the number of arrangements of all the six letters in the word PEPPER?
15. How may permutations of $\{a,b,c,d,e,f,g\}$ and with $a$ ?
16. How many different bit strings are there of length seven?
17. Show that $C(2n,2)=2C(n,2)+n^2$.

# Unit – III (Graph Theory)

1. Define Pseudo graph.
2. Define complete graph and give an example.
3. Define a regular graph. Can a complete graph be a regular graph?
4. When is a simple graph G bipartite? Give an example.
5. Define a connected graph and a disconnected graph with examples.
6. Define strongly connected graph.
7. Is the directed graph given below strongly connected? Why or why not?




8. Define complete bipartite graph.
9. Draw a complete bipartite graph of $K_{2,3}$ and $K_{3,3}$.
10. Define isomorphism of two graphs.
11. Define self complementary graph.
12. Give an example of an Euler graph.
13. Give an example of a non-Eulerian graph which is Hamiltonian.

14. Give an example of a graph which is Eulerian but not Hamiltonian.

15. State the handshaking theorem.

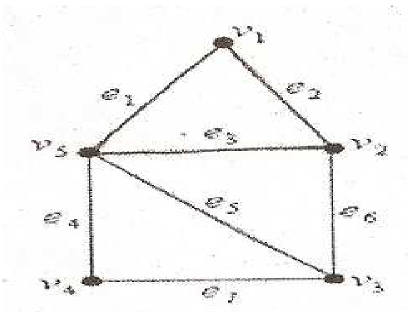
16. State the necessary and sufficient conditions for the existence of an Eulerian path in a connected graph.

17. Define complementary graph $\bar{G}$ of a simple graph $G$. If the degree sequence of the simple graph is 4, 3, 3, 2, 2, what is the degree sequence of $\bar{G}$.

18. For which value of *m* and *n* does the complete bipartite graph $K_{m,n}$ have an (i) Euler circuit (tour) (ii) Hamilton circuit (cycle).

19. Draw the graph represented by the given adjacency matrix $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$.

20. Obtain the adjacency matrix of the graph given below.



## Unit – IV (Algebraic Structures)

1. Define a semigroup.
2. Define monoids.
3. State any two properties of a group.
4. Prove that identity element is unique in a group.
5. When is a group $(G,*)$ called abelian?
6. Prove that every subgroup of an abelian group is normal.
7. Prove that the identity of a subgroup is the same as that of the group.

8. Define homomorphism and isomorphism between two algebraic systems.

9. Give an example for homomorphism.

10. If $a$ and $b$ are any two elements of a group $\langle G,* \rangle$, show that $G$ is an abelian group if and only if $(a*b)^2 = a^2 * b^2$.

11. State Lagrange's theorem in group theory.

12. Show that every cyclic group is abelian.

13. Let $\langle M,*,e_M \rangle$ be a monoidand $a \in M$. If $a$ invertible, then show that its inverse is unique.

14. If $'a'$ is a generator of a cyclic group G, then show that $a^{-1}$ is also a generator of G.

15. Show that the set of all elements $a$ of a group $(G,*)$ such that $a*x = x*a$ for every $x \in G$ is a subgroup of $G$.

16. Obtain all the distinct left – cosets of $\{ [0],[3] \}$ in the group $(Z_6, +_6)$ and find their union.

17. Define ring and give an example.

18. Define a field in an algebraic system.

19. Give an example of a ring which is not a field.

20. Define a commutative ring.

# Unit – V (Lattices and Boolean algebra)

1. Draw the Hasse diagram of $\langle X, \leq \rangle$, where $X = \{2,4,5,10,12,20,25\}$ and the relation $\leq$ be such that $x \leq y$ is $x$ and $y$.

2. Let $X = \{1,2,3,4,6,8,12,24\}$ and $R$ be a division relation. Find the Hasse diagram of the poset $\langle X, R \rangle$.

3. Let $A = \{a,b,c\}$ and $\rho(A)$ be its power set. Draw a Hasse diagram of $\langle \rho(A), \subseteq \rangle$.

4. Show that least upper bound of a subset $B$ in a poset $(A, \leq)$ is unique is it exists.

5. Define a lattice. Give suitable example.

6. Define sub-lattice.
7. When a lattice is called complete?
8. When is a lattice said to be bounded?
9. Define lattice homomorphism.
10. Show that in a distributive lattice, if complement of an element exits then it must be unique.
11. Give an example of a distributive lattice but not complemented.
12. In a Lattice $(L,\leq)$, prove that $a\wedge(a\vee b)=a$, for all $a,b\in L$.
13. Show that in a lattice if $a\leq b\leq c$, then $a\oplus b=b*c$ and $(a*b)\oplus(b*c)=(a\oplus b)*(b\oplus c)$.
14. Check whether the posets $\{(1,3,6,9),D\}$ and $\{(1,5,25,125),D\}$ are lattices or not. Justify your claim.
15. Define a Boolean algebra.
16. When is a lattice said to be a Boolean algebra?
17. Is there a Boolean algebra with five elements? Justify your answer.
18. Prove the Boolean identity: $a.b+a.b'=a$.
19. Show that in a Boolean algebra $ab'+a'b=0$ if and only if $a=b$.
20. Show that the absorption laws are valid in a Boolean algebra.

---- *All the Best*----