SUBJECT NAME : **Discrete Mathematics**

SUBJECT CODE : **MA 6566**

MATERIAL NAME : **Part – A questions**

REGULATION : **R2013**

UPDATED ON : **April-May 2018**

(Scan the above Q.R code for the direct download of this material)

Unit – I (Logic and Proofs)

1. Define Proposition.
2. Define Tautology with an example.
3. Define a rule of Universal specification.
4. Find the truth table for the statement $P \to Q$.
5. Construct the truth table for $P \to \neg Q$.
6. Construct a truth table for the compound proposition $(p \to q) \to (q \to p)$.
7. Using truth table show that $P \vee (P \wedge Q) \equiv P$.
8. Construct the truth table the compound proposition $(p \to q) \leftrightarrow (\neg p \to \neg q)$.
9. Express $A \leftrightarrow B$ in terms of the connectives $\{\wedge, \neg\}$.
10. Given $P = \{2,3,4,5,6\}$, state the truth value of the statement $(\exists x \in P)(x + 3 = 10)$.
11. Show that the propositions $p \to q$ and $\neg p \vee q$ are logically equivalent.
12. Show that $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent.
13. Prove that $p,\ p \to q,\ q \to r \ \Rightarrow\ r$.
14. Without using truth table show that $P \to (Q \to P) \Rightarrow \neg P \to (P \to Q)$.
15. Show that $(P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$ is a tautology.

16. Using truth table, show that the proposition $p \vee \neg(p \wedge q)$ is a tautology.

17. Is $(\neg p \wedge (p \vee q)) \rightarrow q$ a tautology?

18. Write the negation of the statement $(\exists x)(\forall y)\, p(x, y)$.

19. What are the negations of the statements $\forall x(x^2 > x)$ and $\exists x(x^2 = 2)$?

20. Give an indirect proof of the theorem "If $3n+2$ is odd, then n is odd".

21. Let $P(x)$ denote the statement $x \leq 4$. Write the truth values of $P(2)$ and $P(6)$.

22. When do you say that two compound propositions are equivalent?

23. What are the contra positive, the converse and the inverse of the conditional statement "*If you work hard then you will be rewarded*".

24. What is the contra positive of the statement. "The home team wins whenever it is reining"?

25. Give the contra positive statement of the statement, "If there is rain, then I buy an umbrella".

26. Write the symbolic representation of "if it rains today, then I buy an umbrella".

27. Give the symbolic form of "Some men are giant".

28. Symbolically express the following statement.

    " It is not true that 5 star rating always means good food and good service ".

Unit – II (Combinatorics)

1. State the principle of strong induction.

2. Use mathematical induction to show that $n! \geq 2^{n+1},\ n = 1, 2, 3, \ldots$.

3. Use mathematical induction to show that $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$.

4. If seven colours are used to paint 50 bicycles, then show that at least 8 bicycles will be the same colour.

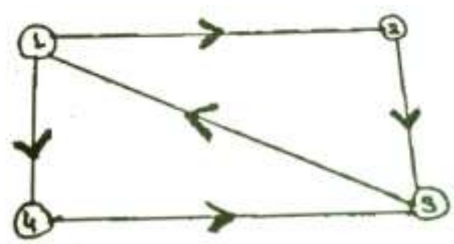
5. Write the generating function for the sequence $1, a, a^2, a^3, a^4, \ldots$.

6. Find the recurrence relation for the Fibonacci sequence.
7. Find the recurrence relation satisfying the equation $y_n = A(3)^n + B(-4)^n$.
8. Solve the recurrence relation $y(k) - 8y(k-1) + 16y(k-2) = 0$ for $k \geq 2$, where $y(2) = 16$ and $y(3) = 80$.
9. Solve: $a_k = 3a_{k-1}$, for $k \geq 1$, with $a_0 = 2$.
10. Find the number of non-negative integer solutions of the equation $x_1 + x_2 + x_3 = 11$.
11. State the Pigeonhole principle.
12. Find the minimum number of students need to guarantee that five of them belongs to the same subject, if there are five different major subjects.
13. What is well ordering principle?
14. In how many ways can all the letters in MATHEMATICAL be arranged.
15. How many different words are there in the word ENGINEERING?
16. How many permutations can be made out of letter or word 'COMPUTER'?
17. How many different words are there in the word MATHEMATICS?
18. What is the number of arrangements of all the six letters in the word PEPPER?
19. How many permutations are there in the word M I S S I S S I P P I ?
20. How many permutations are there on the word "MALAYALAM"?
21. How many permutations of the letters ABCDEFGH contain the string ABC?
22. How may permutations of $\{a, b, c, d, e, f, g\}$ and with a ?
23. How many different bit strings are there of length seven?
24. Show that $C(2n, 2) = 2C(n, 2) + n^2$.

Unit – III (Graph Theory)

1. When is a simple graph G bipartite? Give an example.
2. How many edges are there in a graph with 10 vertices each of degree 3?

3. Define Pseudo graph.
4. Define complete graph and give an example.
5. Define a connected graph and a disconnected graph with examples.
6. How many edges are there in a graph with 10 vertices each of degree 5?
7. Define strongly connected graph.
8. Is the directed graph given below strongly connected? Why or why not?



9. Draw the complete graph K_5.
10. Define a bipartite graph.
11. Draw a complete bipartite graph of $K_{2,3}$ and $K_{3,3}$.
12. Define a regular graph. Can a complete graph be a regular graph?
13. Define isomorphism of two graphs.
14. Define complementary graph.
15. Define self-complementary graph with example.
16. Give an example of an Euler graph.
17. Define Hamiltonian path.
18. Give an example of a non-Eulerian graph which is Hamiltonian.
19. Give an example of a graph which is Eulerian but not Hamiltonian.
20. State the handshaking theorem.
21. Show that there does not exist a graph with 5 vertices with degrees 1, 3, 4, 2, 3 respectively.
22. State the necessary and sufficient conditions for the existence of an Eulerian path in a connected graph.

23. Let G be a graph with ten vertices. If four vertices has degree four and six vertices has degree five, then find the number of edges of G.

24. Define complementary graph $\bar{G}$ of a simple graph G. If the degree sequence of the simple graph is 4, 3, 3, 2, 2, what is the degree sequence of $\bar{G}$.

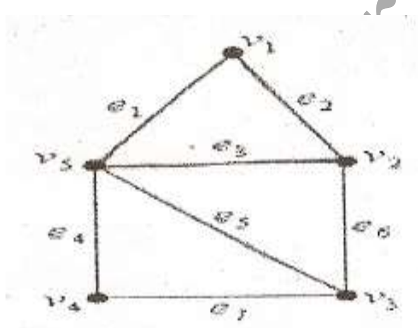
25. For which value of *m* and *n* does the complete bipartite graph $K_{m,n}$ have an (i) Euler circuit (tour) (ii) Hamilton circuit (cycle).

26. How do you find the number of different paths of length r from i to j in a graph G with adjacency matrix A?

27. Draw the graph with the following adjacency matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

28. Draw the graph represented by the given adjacency matrix $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$.

29. Obtain the adjacency matrix of the graph given below.



Unit – IV (Algebraic Structures)

1. State any two properties of a group.

2. Prove that identity element in a group is unique.

3. Define a semi group.

4. Define monoids.

5. Give an example of semi group but not a monoid.

6. When is a group $(G,*)$ called abelian?

7. Find the idempotent elements of $G=\{1,-1,i,-i\}$ under the binary operation multiplication.

8. Prove the in a group idempotent law is true only for identity element.

9. If a subset $S \neq \phi$ of G is a subgroup of $(G,*)$, then prove that for any pair of elements $a,b \in S$, $a*b^{-1} \in S$.

10. Prove that every subgroup of an abelian group is normal.

11. Prove that the identity of a subgroup is the same as that of the group.

12. Define homomorphism and isomorphism between two algebraic systems.

13. Give an example for homomorphism.

14. If a and b are any two elements of a group $\langle G,* \rangle$, show that G is an abelian group if and only if $(a*b)^2 = a^2 * b^2$.

15. State Lagrange's theorem.

16. Show that every cyclic group is abelian.

17. Let $\langle M,*,e_M \rangle$ be a monoid and $a \in M$. If a invertible, then show that its inverse is unique.

18. If $'a'$ is a generator of a cyclic group G, then show that a^{-1} is also a generator of G.

19. Let Z be the group of integers with the binary operation $*$ defined by $a*b=a+b-2$, for all $a,b \in Z$. Find the identity element of the group $\langle Z,* \rangle$.

20. Show that the set of all elements a of a group $(G,*)$ such that $a*x = x*a$ for every $x \in G$ is a subgroup of G.

21. Obtain all the distinct left – cosets of $\{ [0],[3] \}$ in the group $(Z_6,+_6)$ and find their union.

22. Define a ring.

23. Define a field in an algebraic system.

24. Give an example of a ring which is not a field.

25. Define a commutative ring.

Unit – V (Lattices and Boolean algebra)

1. Let $A=\{1,2,5,10\}$ with the relation divides. Draw the Hasse diagram.

2. Draw the Hasse diagram of $\langle X,\leq\rangle$, where $X=\{2,4,5,10,12,20,25\}$ and the relation $\leq$ be such that $x\leq y$ is x and y.

3. Let $A=\{a,b,c\}$ and $\rho(A)$ be its power set. Draw a Hasse diagram of $\langle \rho(A),\subseteq\rangle$.

4. Let $X=\{1,2,3,4,5,6\}$ and R be a relation defined as $\langle x,y\rangle\in R$ if and only if $x-y$ is divisible by 3. Find the elements of the relation R.

5. Give a relation which is both a partial ordering relation and an equivalence relation.

6. Prove that a lattice with five element is not a Boolean algebra.

7. Show that least upper bound of a subset B in a poset $(A,\leq)$ is unique is it exists.

8. Define a lattice. Give suitable example.

9. Define sub-lattice.

10. When a lattice is called complete?

11. When is a lattice said to be bounded?

12. Define lattice homomorphism.

13. Show that in a distributive lattice, if complement of an element exits then it must be unique.

14. Give an example of a distributive lattice but not complemented.

15. In a Lattice $(L,\leq)$, prove that $a\wedge(a\vee b)=a$, for all $a,b\in L$.

16. Show that in a lattice if $a\leq b\leq c$, then $a\oplus b=b*c$ and $(a*b)\oplus(b*c)=(a\oplus b)*(b\oplus c)$.

17. Check whether the posets $\{(1,3,6,9),D\}$ and $\{(1,5,25,125),D\}$ are lattices or not. Justify your claim.

18. When is a lattice said to be a Boolean algebra?

19. Define a Boolean algebra.

20. Give an example of a two elements Boolean algebra.

21. State the De Morgan's laws in a Boolean algebra.

22. Is there a Boolean algebra with five elements? Justify your answer.

23. Show that the absorption laws are valid in a Boolean algebra.

24. Prove the Boolean identity: $a.b + a.b' = a$.

25. Show that in a Boolean algebra $ab' + a'b = 0$ if and only if $a = b$.

---- *All the Best* ----