B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2012

Fifth Semester

Computer Science and Engineering

MA2265 – DISCRETE MATHEMATICS

(Regulation 2008)

Time : Three hours Maximum : 100 Marks

Answer ALL questions

PART A – (10 x 2 = 20 Marks)

1. Using truth table, show that the proposition $p \vee \neg(p \wedge q)$ is a tautology.
2. Write the negation of the statement $(\exists x)(\forall y)\, p(x,y)$.
3. Find the number of non-negative integer solutions of the equation $x_1 + x_2 + x_3 = 11$.
4. Find the recurrence relation for the Fibonacci sequence.
5. Define isomorphism of two graphs.
6. Give an example of an Euler graph.
7. Define a semigroup.
8. If $'a'$ is a generator of a cyclic group G, then show that a^{-1} is also a generator of G.
9. In a Lattice $(L,\leq)$, prove that $a \wedge (a \vee b) = a$, for all $a, b \in L$.
10. Define a Boolean algebra.

PART B – (5 x 16 = 80 Marks)

11. (a) (i) Prove that the following argument is valid: $p \to \neg q,\ r \to q,\ r \Rightarrow \neg p$.

(ii) Determine the validity of the following argument:

If 7 is less then 4, then 7 is not a prime number, 7 is not less then 4. Therefore 7 is a prime number.

Or

(b) (i) Verify the validity of the following argument. Every living thing is a plant or an animal. John's gold fish is alive and it is not a plant. All animals have hearts. Therefore John's gold fish has a heart.

(ii) Show that $(\forall x)(P(x) \to Q(x)),\ (\exists y)P(y) \Rightarrow (\exists x)Q(x)$.

12. (a) (i) Prove by the principle of mathematical induction, for $'n'$ a positive integer,

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$ (10)

(ii) Find the number of distinct permutations that can be formed from all the letters of each word (1) RADAR (2) UNUSUAL. (6)

Or

(b) Solve the recurrence relation, $S(n) = S(n-1) + 2(n-1)$, with $S(0) = 3,\ S(1) = 1$, by finding its generating function.

13. (a) Prove that a connected graph G is Eulerian if and only if all the vertices are on even degree.

Or

(b) Show that graph G is disconnected if and only if its vertex set V can be partitioned into two nonempty subsets V_1 and V_2 such that there exists no edge is G whose one end vertex is in V_1 and the other in V_2.

14. (a) Let $f : G \to G'$ be a homorphism of groups with Kernel K. Then prove that K is a normal subgroup of G and G / K is isomorphic to the image of f.

Or

(b) State and prove Lagrange's theorem.

15. (a) Show that the direct product of any two distributive lattices is a distributive lattice.

Or

(b) Let $\boldsymbol{B}$ be a finite Boolean algebra and let $\boldsymbol{A}$ be the set of all atoms of $\boldsymbol{B}$. Then prove that the Boolean algebra $\boldsymbol{B}$ is isomorphic to the Boolean algebra $\boldsymbol{P(A)}$, where $\boldsymbol{P(A)}$ is the power set of $\boldsymbol{A}$.