B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2010

Fifth Semester

Computer Science and Engineering

MA2265 – DISCRETE MATHEMATICS

(Regulation 2008)

Time : Three hours | Maximum : 100 Marks

Answer ALL questions

PART A – (10 x 2 = 20 Marks)

1. When do you say that two compound propositions are equivalent?
2. Prove that $p, p \rightarrow q, q \rightarrow r \Rightarrow r$.
3. State Pigeonhole principle.
4. Find the recurrence relation satisfying the equation $y_n = A(3)^n + B(-4)^n$.
5. Define strongly connected graph.
6. State the necessary and sufficient conditions for the existence of an Eulerian path in a connected graph.
7. State any two properties of a group.
8. Define a commutative ring.
9. Define Boolean algebra.
10. Define sub-lattice.

PART B – (5 x 16 = 80 Marks)

11. (a) (i) Prove that the premises $a \to (b \to c),\ d \to (b \wedge \neg c)$ and $(a \wedge d)$ are inconsistent.

(ii) Obtain the principal disjunctive normal form and principal conjunction form of the statement

$$p \vee (\neg p \to (q \vee (\neg q \to r))).$$

Or

(b) (i) Prove that $\forall x(P(x) \to Q(x)),\ \forall x(R(x) \to \neg Q(x)) \Rightarrow \forall x(R(x) \to \neg P(x))$.

(ii) Without using the truth table, prove that $\neg p \to (q \to r) \equiv q \to (p \vee r)$.

12. (a) (i) Prove, by mathematical induction, that for all $n \geq 1,\ n^3 + 2n$ is a multiple of 3.

(ii) Using the generating function, solve the difference equation

$$y_{n+2} - y_{n+1} - 6y_n = 0,\ y_1 = 1,\ y_0 = 2.$$

Or

(b) (i) How many positive integers n can be formed using the digits 3,4,4,5,5,6,7 if n has to exceed 5000000?

(ii) Find the number of integers between 1 and 250 both inclusive that are divisible by any of the integers 2,3,5,7.

13. (a) (i) Draw the complete graph K_5 with vertices A, B, C, D, E. Draw all complete sub graph of K_5 with 4 vertices.

(ii) If all the vertices of an undirected graph are each of degree k, show that the number of edges of the graph is a multiple of k.

Or

(b) (i) Draw the graph with 5 vertices, A, B, C, D, E such that $\deg(A) = 3$, B is an odd vertex, $\deg(C) = 2$ and D and E are adjacent.

(ii) The adjacency matrices of two pairs of graph as given below. Examine the isomorphism of G and H by finding a permutation matrix. $A_G = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $A_H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

14. (a) (i) If $(G,*)$ is an abelian group, show that $(a*b)^2 = a^2 * b^2$.

(ii) Show that $(Z,+,\times)$ is an integral domain where Z is the set of all integers.

Or

(b) (i) State and prove Lagrange's theorem.

(ii) If $(Z,+)$ and $(E,+)$ where Z is the set all integers and E is the set all even integers, show that the two semi groups $(Z,+)$ and $(E,+)$ are isomorphic.

15. (a) (i) Show that $(N,\leq)$ is a partially ordered set where N is set of all positive integers and $\leq$ is defined by $m \leq n$ iff $n-m$ is a non-negative integer.

(ii) In a Boolean algebra, prove that $(a \wedge b)' = a' \vee b'$.

Or

(b) (i) In a Lattice $(L,\leq)$, prove that $x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$.

(ii) If S_{42} is the set all divisors of 42 and D is the relation "divisor of" on S_{42}, prove that $\{S_{42}, D\}$ is a complemented Lattice.