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Name of the Student: Branch:

Unit – I (Ordinary Differential Equation)

• ODE with Constant Coefficients

1. Solve the equation $\left(D^2+4\right)y = x^2\cos 2x$. (M/J 2009),(N/D 2011)
2. Solve the equation $\left(D^2-3D+2\right)y = 2\cos\left(2x+3\right)+2e^x$. (N/D 2009)
3. Solve $\left(D^2+16\right)y = \cos^3 x$. (N/D 2010)
4. Solve : $\left(D^2+3D+2\right)y = \sin x + x^2$. (M/J 2011)
5. Solve the equation $\left(D^2+5D+4\right)y = e^{-x}\sin 2x$. (A/M 2011),(ND 2012)
6. Solve the equation $\left(D^2+4D+3\right)y = e^{-x}\sin x$. (M/J 2010)
7. Solve: $\left(D^2-4D+3\right)y = e^x\cos 2x$. (M/J 2012)
8. Solve $\left(D^2+4D+3\right)y = 6e^{-2x}\sin x\sin 2x$. (N/D 2011)
9. Solve $\left(D^3-7D-6\right)y = \left(1+x\right)e^{2x}$. (N/D 2014)
10. Solve $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+y = 8xe^x\sin x$. (N/D 2013)

- # Method of Variation of Parameters

1. Solve $\left(D^2+a^2\right)y=\tan ax$ by the method of variation of parameters. (M/J 2009)

2. Solve $\frac{d^2y}{dx^2}+a^2y=\tan ax$ by method of variation of parameters. (M/J 2011)

3. Solve $y''+a^2y=\tan ax$ by variation of parameters method. (N/D 2014)

4. Solve $2\frac{d^2y}{dx^2}+8y=\tan 2x$ by method of variation of parameters. (N/D 2013)

5. Apply method of variation of parameters to solve $\left(D^2+4\right)y=\cot 2x$.
   (N/D 2009),(N/D 2011)

6. Solve $\left(D^2+a^2\right)y=\sec ax$ using the method of variation of parameters.(M/J 2012)

7. Solve $\frac{d^2y}{dx^2}+y=\cos ecx$ by the method of variation of parameters.
   (A/M 2011),(ND 2012)

8. Solve $\left(D^2+1\right)y=x\sin x$ by the method of variation of parameters. (M/J 2010)

9. Using variation of parameters, solve $\left(2D^2-D-3\right)y=25e^{-x}$. (N/D 2011)

10. Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=\frac{e^{-x}}{x^2}$ by the method of variation of parameters.(M/J 2013)

- # Cauchy and Legendre Equations

1. Solve the equation $\left(x^2D^2+3xD+5\right)y=x\cos\left(\log x\right)$. (M/J 2009)

2. Solve $\left(x^2D^2-3xD+4\right)y=x^2\cos\left(\log x\right)$. (N/D 2010)

3. Solve $\left(x^2D^2-xD+4\right)y=x^2\sin\left(\log x\right)$. (M/J 2012),(N/D 2009)

4. Solve $\left(x^2D^2-2xD-4\right)y=x^2+2\log x$. (M/J 2010)

5. Solve the equation $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}=\frac{12\log x}{x^2}$. (N/D 2012)

6. Solve $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=x^2\ln x$. (N/D 2011)

7. Solve $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=x^2+\frac{1}{x^2}$. (M/J 2013)

8. Solve $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+y=e^{e^{\log x}}$. (N/D 2013)

9. Solve $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin\left[\log(1+x)\right]$. (A/M 2011)

10. Solve $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos\left[\log(1+x)\right]$. (N/D 2011)

11. Solve $\left[(x+1)^2D^2+(x+1)D+1\right]y=4\cos\left[\log(x+1)\right]$. (N/D 2014)

12. Solve $\left(3x+2\right)^2\frac{d^2y}{dx^2}+3\left(3x+2\right)\frac{dy}{dx}-36y=3x^2+4x+1$. (M/J 2013)

- **Simultaneous Differential Equations**

1. Solve $\frac{dx}{dt}+y=\sin t,\ x+\frac{dy}{dt}=\cos t$ given $x=2$ and $y=0$ at $t=0$. (M/J 2009)

2. Solve $\frac{dx}{dt}+2y=\sin 2t,\ \frac{dy}{dt}-2x=\cos 2t$. (M/J 2012),(N/D 2009)

3. Solve $\frac{dx}{dt}+2y=-\sin t,\ \frac{dy}{dt}-2x=\cos t$ given $x=1$, $y=0$ at $t=0$. (N/D 2010)

4. Solve $\frac{dx}{dt}-y=t$ and $\frac{dy}{dt}+x=t^2$. (A/M 2011)

5. Solve $\frac{dx}{dt}-y=t$ and $\frac{dy}{dt}+x=t^2$ given $x(0)=y(0)=2$. (N/D 2011)

6. Solve $\frac{dx}{dt}+y=e^t$, $x-\frac{dy}{dt}=t$. (N/D 2012)

7. Solve $\frac{dx}{dt}+2x+3y=2e^{2t}, \frac{dy}{dt}+3x+2y=0.$ (M/J 2010)

8. Solve $\frac{dx}{dt}+5x-2y=t, \frac{dy}{dt}+2x+y=0.$ (M/J 2013)

9. Solve $\frac{dx}{dt}+2x-3y=t$ and $\frac{dy}{dt}-3x+2y=e^{2t}$. (N/D 2011),(N/D 2014)

10. Solve $\frac{dx}{dt}+4x+3y=t$ and $\frac{dy}{dt}+2x+5y=e^{2t}$. (N/D 2013)

Unit – II (Vector Calculus)

• Simple problems on vector calculus

1. Find the directional derivative of $\phi=2xy+z^2$ at the point $(1,-1,3)$ in the direction of $\vec{i}+2\vec{j}+2\vec{k}$. (M/J 2009)

2. Prove that $\vec{F}=(6xy+z^3)\vec{i}+(3x^2-z)\vec{j}+(3xz^2-y)\vec{k}$ is irrotational vector and find the scalar potential such that $\vec{F}=\nabla\varphi$. (M/J 2010)

3. Show that $\vec{F}=(y^2+2xz^2)\vec{i}+(2xy-z)\vec{j}+(2x^2z-y+2z)\vec{k}$ is irrotational and hence find its scalar potential. (M/J 2012),(N/D 2014)

4. Show that $\vec{F}=(2xy-z^2)\vec{i}+(x^2+2yz)\vec{j}+(y^2-2zx)\vec{k}$ is irrotational and find its scalar potential. (N/D 2012)

5. Show that $\vec{F}=(x^2+xy^2)\vec{i}+(y^2+x^2y)\vec{j}$ is irrotational and find its scalar potential. (N/D 2013)

6. Find the angle between the normals to the surface $xy^3z^2=4$ at the points $(-1,-1,2)$ and $(4,1,-1)$. (M/J 2009)

7. Find the angle between the normals to the surface $xy=z^2$ at the points $(1,4,2)$ and $(-3,-3,3)$. (A/M 2011)

8. Find a and b so that the surfaces $ax^3 - by^2z - (a+3)x^2 = 0$ and $4x^2y - z^3 - 11 = 0$ cut orthogonally at the point $(2,-1,-3)$. (N/D 2013)

9. Find the work done in moving a particle in the force field given by

   $\vec{F} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$ along the straight line from $(0,0,0)$ to $(2,1,3)$.
   (M/J 2012)

10. If $\vec{r}$ is the position vector of the point (x, y, z), Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$.

    (N/D 2010)

11. Determine $f(r)$, where $\vec{r} = xi + yj + zk$, if $f(r)\vec{r}$ is solenoidal and irrotational.

    (N/D 2011)

12. If $\vec{F}$ is a vector point function, prove that $curl\left(curl F^{-1}\right) = \nabla\left(\nabla \bullet F^{-1}\right) - \nabla^2 \vec{F}$.

    (N/D 2011)(AUT)

13. Prove that $\mathbf{curl}\left(\vec{u}\times\vec{v}\right) = \left(\vec{v}\cdot\nabla\right)\vec{u} - \left(\vec{u}\cdot\nabla\right)\vec{v} + \vec{u}\,\mathbf{div}\,\vec{v} - \vec{v}\,\mathbf{div}\,\vec{u}$. (N/D 2009)

14. Evaluate $\int_C \left(x^2 + xy\right)dx + \left(x^2 + y^2\right)dy$ where C is the square bounded by the

    lines $x = 0, x = 1, y = 0$ **and** $y = 1$. (N/D 2009),(N/D 2011)

15. Evaluate $\iint_s \vec{F}\bullet\vec{n}\, ds$ where $\vec{F} = 2xy\vec{i} + yz^2\vec{j} + xz\vec{k}$ and S is the surface of the

    parallelepiped bounded by $x = 0, y = 0, z = 0, x = 2, y = 1$ and $z = 3$. (M/J 2011)

- **Green's Theorem**

1. Verify Green's theorem in a plane for $\int_C \left[\left(3x^2 - 8y^2\right)dx + \left(4y - 6xy\right)dy\right]$, Where C is the boundary of the region defined by the lines $x = 0, y = 0$ and $x + y = 1$.
   (N/D 2010) ,(A/M 2011),(M/J 2011), (M/J 2012)

2. Verify Green's theorem for $\int_C \left(3x^2 - 8y^2\right)dx + \left(4y - 6xy\right)dy$ where C is the boundary of the region defined by $x = y^2,\ y = x^2$. (M/J 2010)

3. Verify Green's theorem for $\vec{V} = \left(x^2 + y^2\right)\vec{i} - 2xy\vec{j}$ taken around the rectangle bounded by the lines $x = \pm a,\ y = 0$ and $y = b$. (N/D 2012)

- **Stoke's Theorem**

1. Verify Stoke's theorem for $\vec{F} = \left(x^2 + y^2\right)\vec{i} - 2xy\vec{j}$ taken around the rectangle formed by the lines $x = -a,\ x = a,\ y = 0$ and $y = b$. (N/D 2013)

2. Verify Stoke's theorem for $\vec{F} = \left(x^2 + y^2\right)\vec{i} + 2xy\vec{j}$ where S is the rectangle in the xy-plane formed by the lines $x = 0,\ x = a,\ y = 0$ and $y = b$. (N/D 2014)

3. Verify Stoke's theorem for $\vec{F} = xy\vec{i} - 2yz\vec{j} - zx\vec{k}$ where S is the open surface of the rectangular parallelepiped formed by the planes $x = 0, x = 1, y = 0, y = 2$ and $z = 3$ above the XY plane. (M/J 2009)

4. Verify Stoke's thorem for the vector $\vec{F} = (y - z)i + yzj - xzk$, where S is the surface bounded by the planes $x = 0, y = 0, z = 0, x = 1, y = 1, z = 1$ and C is the square boundary on the xoy-plane. (N/D 2011)

5. Verify Stoke's theorem when $\vec{F} = \left(2xy - x^2\right)\vec{i} - \left(x^2 - y^2\right)\vec{j}$ and C is the boundary of the region enclosed by the parabolas $y^2 = x$ and $x^2 = y$. (N/D 2009)

6. Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ over the upper half surface $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy-plane. (M/J 2013)

7. Evaluate $\int_C \left(\sin z dx - \cos x dy + \sin y dz\right)$ by using Stoke's theorem, where C is the boundary of the rectangle defined by $0 \le x \le \pi,\ 0 \le y \le 1,\ z = 3$. (N/D 2009)

8. Using Stokes theorem, evaluate $\int_C \vec{F} \bullet d\vec{r}$, where $\vec{F} = y^2\vec{i} + x^2\vec{j} - (x + z)\vec{k}$ and 'C' is the boundary of the triangle with vertices at $(0,0,0), (1,0,0), (1,1,0)$. (M/J 2012)

9. Using Stoke's theorem prove that $\textbf{curl grand } \phi = 0$. (M/J 2011)

Gauss Divergence Theorem

1. Verify Gauss divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ where S is the surface of the cuboid formed by the planes $x = 0, x = a, y = 0, y = b, z = 0$ and $z = c$.
(M/J 2009),(N/D 2014)

2. Verify Gauss Divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.
(N/D 2010),(A/M 2011),(N/D 2012),(N/D 2013)

3. Verify Gauss – divergence theorem for the vector function $\vec{f} = \left(x^3 - yz\right)\vec{i} - 2x^2y\vec{j} + 2\vec{k}$ over the cube bounded by $x = 0, y = 0, z = 0$ and $x = a, y = a, z = a$. (M/J 2010),(N/D 2011)

4. Verify divergence theorem for $\vec{F} = x^2\vec{i} + z\vec{j} + yz\vec{k}$ over the cube formed by the planes $x = \pm 1,\ y = \pm 1,\ z = \pm 1$. (M/J 2013)

5. Verify Gauss's theorem for $\vec{F} = \left(x^2 - yz\right)i + \left(y^2 - zx\right)j + \left(z^2 - xy\right)k$ over the rectangular parallelepiped formed by $0 \le x \le 1, 0 \le y \le 1$ and $0 \le z \le 1$.
(N/D 2011)(AUT)

Unit – III (Analytic Function)

Harmonic Function & Analytic Function

1. Verify that the families of curves $u = c_1$ and $v = c_2$ cut orthogonally, when $u + iv = z^3$.
(N/D 2009)

2. Prove that $u = e^{-y}\cos x$ and $v = e^{-x}\sin y$ satisfy Laplace equations, but that $u + iv$ is not an analytic function of z. (M/J 2011)

3. When the function $f(z) = u + iv$ is analytic, prove that the curves $u = \text{constant}$ and $v = \text{constant}$ are orthogonal. (N/D 2009)

4. Show that the families of curves $r^n = a\sec n\theta$ and $r^n = b\operatorname{cosec} n\theta$ cut orthogonally.
(M/J 2011)

5. Show that $u=\frac{1}{2}\log\left(x^2+y^2\right)$ is harmonic. Determine its analytic function. Find also its conjugate. (A/M 2011)

6. Prove that $u=e^{-2xy}\sin\left(x^2-y^2\right)$ is harmonic. Find the corresponding analytic function and the imaginary part. (N/D 2013)

7. Prove that $u=x^2-y^2$ and $v=\frac{-y}{x^2+y^2}$ are harmonic but $u+iv$ is not regular. (N/D 2010)

8. Prove that every analytic function $w=u+iv$ can be expressed as a function z alone, not as a function of $\bar{z}$. (M/J 2010),(M/J 2012)

9. Find the analytic function $f(z)=P+iQ$, if $P-Q=\frac{\sin 2x}{\cosh 2y-\cos 2x}$.(M/J 2009)

10. Determine the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y-\cos 2x}$. (N/D 2012)

11. If $w=f(z)$ is analytic, prove that $\frac{dw}{dz}=\frac{\partial w}{\partial x}=-i\frac{\partial w}{\partial y}$. (A/M 2011)

12. Find the analytic function $u+iv$, if $u=\left(x-y\right)\left(x^2+4xy+y^2\right)$. Also find the conjugate harmonic function v. (N/D 2009)

13. Find the analytic function $w=u+iv$ when $v=e^{-2y}\left(y\cos 2x+x\sin 2x\right)$ and find u. (N/D 2011)

14. Find the analytic function $f=u+iv$ given that $u(x,y)=e^{2x}\left(x\sin 2y+y\cos 2y\right)$. (N/D 2014)

15. Prove that $u=e^{x}(x\cos y-y\sin y)$ is harmonic (satisfies Laplace's equation) and hence find the analytic function $f(z)=u+iv$. (N/D 2010),(M/J 2013)

16. If $f(z)$ is a analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left|f(z)\right|^2=4\left|f'(z)\right|^2$. (M/J 2009), (A/M 2011),(M/J 2013),(N/D 2014)

17. If $f(z)$ is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log|f(z)|=0$. (M/J 2012)

18. If $f(z)$ is analytic function of z in any domain, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^p = p^2|f'(z)|^2|f(z)|^{p-2}.$$ (N/D 2011)(AUT)

• Conformal Mapping

1. Find the image of the half plane $x>c$, when $c>0$ under the transformation $w=\frac{1}{z}$. Show the regions graphically. (M/J 2009),(N/D 2012)

2. Find the image of the circle $|z-1|=1$ under the mapping $w=\frac{1}{z}$. (N/D 2009)

3. Find the image of the circle $|z-2i|=2$ under the transformation $w=\frac{1}{z}$. (M/J 2013)

4. Find the image of the hyperbola $x^2-y^2=1$ under the transformation $w=\frac{1}{z}$. (M/J 2010),(M/J 2012),(N/D 2012)

5. Find the image of $|z|=2$ under the mapping (1) $w=z+3+2i$ (2) $w=3z$. (A/M 2011)

6. Prove that the transformation $w=\frac{z}{1-z}$ maps the upper half of z - plane on to the upper half of w - plane. What is the image of $|z|=1$ under this transformation? (M/J 2010),(N/D 2012),(N/D 2013)

7. Prove that the transformation $w=\frac{1}{z}$ maps the family of circles and straight lines into the family of circles or straight lines. (N/D 2011)

8. Show that the transformation $w=\frac{1}{z}$ transforms, in general, circles and straight lines into circles and straight lines that are transformed into straight lines and circles respectively. (N/D 2011)(AUT)

• Bilinear Transformation

1. Find the bilinear transformation which maps the points $z=0,-i,-1$ into w – plane $w=i,1,0$ respectively. (M/J 2009)

2. Find the bilinear transformation which maps the points $z=0,1,\infty$ into $w=i,1,-i$ respectively. (M/J 2010),(M/J 2012),(M/J 2013)

3. Find the bilinear transformation that maps the points $z=\infty,i,0$ onto $w=0,i,\infty$ respectively. (N/D 2012)

4. Find the bilinear transformation which maps the points $\infty,2,-1$ to $1,\infty,0$ respectively. (N/D 2014)

5. Find the bilinear map which maps the points $z=0,-1,i$ onto points $w=i,0,\infty$. Also find the image of the unit circle of the z plane. (N/D 2013)

6. Find the bilinear transformation which maps the points $z=1,i,-1$ into the points $w=i,0,-i$. Hence find the image of $|z|<1$. (M/J 2011)

7. Find the bilinear transformation that transforms $1,\ i$ and -1 of the z – plane onto $0,\ 1$ and ∞ of the w – plane. Also show that the transformation maps interior of the unit circle of the z – plane on to upper half of the w – plane. (N/D 2010)

8. Find the Bilinear transformation that maps the points $1+i,-i,2-i$ of the z - plane into the points $0,1,i$ of the w - plane. (N/D 2011)

Unit – IV (Complex Integration)

• Cauchy Integral Formula and Cauchy Residu Theorem

1. Evaluate $\int_c \frac{z\,dz}{(z-1)(z-2)^2}$ where C is the circle $|z-2|=\frac{1}{2}$ using Cauchy's integral formula. (M/J 2009),(N/D 2009),(M/J 2012)

2. Evaluate $\int_C \frac{z+1}{(z^2+2z+4)^2}dz$ where C is $|z+1+i|=2$ using Cauchy's integral formula. (A/M 2011),(N/D 2013)

3. Evaluate $\int_c \frac{z+4}{z^2+2z+5}\,dz$, where C is the circle $|z+1+i|=2$, using Cauchy's integral formula. (N/D 2010),(N/D 2011),(N/D 2012)

4. Using Cauchy's integral formula evaluate $\int_c \frac{z}{z^2+1}dz$, where C is the circle $|z+i|=1$. (M/J 2011)

5. Using Cauchy's integral formula, evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)}dz$, Where '$C$' is the circle $|z|=\frac{3}{2}$. (M/J 2010)

6. Evaluate $\int_C \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}\,dz$, where C is $|z|=3$. (N/D 2011),(M/J 2013)

7. Evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)}dz$, where C is the circle $|z-i|=2$ using Cauchy's residue theorem.

- **Contour Integral of Types – I ,II &III**

1. Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ using contour integration.

(N/D 2009), (M/J 2010), (N/D 2010) ,(A/M 2011)

2. Evaluate $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}$ $(a>b>0)$, using contour integration. (N/D 2011),(N/D 2014)

3. Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta}d\theta$, using contour integration. (N/D 2013)

4. Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta}d\theta$ using contour integration. (M/J 2013)

5. Evaluate $\int_0^{2\pi} \frac{\sin^2\theta}{a+b\cos\theta}d\theta,\ a>b>0$. (N/D 2012)

6. Evaluate $\int_0^{2\pi} \frac{d\theta}{1-2x\sin\theta + x^2}$, $(0<x<1)$. (M/J 2009)

7. Evaluate, by contour integration, $\int_0^{2\pi} \frac{d\theta}{1-2a\sin\theta + a^2}$, $0<a<1$. (M/J 2011)

8. Evaluate $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9}\, dx$ using contour integration.

(M/J 2010),(A/M 2011),(N/D 2013)

9. Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{\left(x^2+a^2\right)\left(x^2+b^2\right)}$, $a>b>0$. (M/J 2009),(M/J 2013),(N/D 2014)

10. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{\left(x^2+1\right)\left(x^2+4\right)}$ using contour integration. (N/D 2010)

11. Evaluate using contour integration $\int_{-\infty}^{\infty} \frac{x^2}{\left(x^2+1\right)^2} dx$. (N/D 2011)

12. Evaluate $\int_0^{\infty} \frac{dx}{\left(x^2+a^2\right)^3}$, $a>0$ using contour integration. (N/D 2009)

13. Evaluate $\int_0^{\infty} \frac{\cos mx}{x^2+a^2} dx$, using contour integration. (M/J 2012)

14. Evaluate $\int_{-\infty}^{\infty} \frac{\cos x\, dx}{\left(x^2+a^2\right)\left(x^2+b^2\right)}$, $a>b>0$. (N/D 2011)

- ## Taylor's and Laurent's Series

1. Expand $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ as a Laurent's series in the region $2<|z|<3$.

(A/M 2011),(M/J 2011),(N/D 2011)

2. Find the Laurent's series of $f(z) = \frac{z^2-1}{z^2+5z+6}$ valid in $2<|z|<3$.(M/J 2009)

3. Expand the function $f(z)=\dfrac{z^2-1}{z^2+5z+6}$ in Laurent's series for $|z|>3$.(M/J 2013)

4. Evaluate $f(z)=\dfrac{1}{(z+1)(z+3)}$ in Laurent series valid for the regions $|z|>3$ and $1<|z|<3$. (N/D 2009),(M/J 2012)

5. Find the Laurants's series expansion of $f(z)=\dfrac{1}{z(1-z)}$ valid in the regions $|z+1|<1,\ 1<|z+1|<2$ and $|z+1|>2$. (N/D 2011)

6. Find the Laurent's series of $f(z)=\dfrac{3z-2}{z(z^2-4)}$ valid in the region $2<|z+2|<4$. (N/D 2014)

7. Find the Laurent's series of $f(z)=\dfrac{7z-2}{z(z+1)(z+2)}$ in $1<|z+1|<3$. (M/J 2010)

8. Find the residues of $f(z)=\dfrac{z^2}{(z-1)^2(z+2)^2}$ at its isolated singularities using Laurent's series expansions. Also state the valid region. (N/D 2010),(N/D 2012)

9. Find the residues of $f(z)=\dfrac{z^2}{(z+2)(z-1)^2}$ at its isolated singularities using Laurentz's series expansion. (N/D 2013)

Unit – V (Laplace Transform)

- **Laplace Transform of Periodic Function**

1. Find the Laplace transform of $f(t)=\begin{cases} t, & \text{for } 0<t<a \\ 2a-t, & \text{for } a<t<2a \end{cases}$, $f(t+2a)=f(t)$.
(M/J 2009),(N/D 2009),(A/M 2011),(N/D 2014)

2. Find the Laplace transform of the following triangular wave function given by $f(t)=\begin{cases} t, & 0\le t\le \pi \\ 2\pi-t, & \pi\le t\le 2\pi \end{cases}$ and $f(t+2\pi)=f(t)$. (M/J 2010),(M/J 2012)

3. Find the Laplace transform of $f(t)=\begin{cases} t, & 0<t<1 \\ 0, & 1<t<2 \end{cases}$ and $f(t+2)=f(t)$ for $t>0$.

(N/D 2011)(AUT)

4. Find the Laplace transform of square wave function defined by

$f(t)=\begin{cases} 1, & \text{in } 0<t<a \\ -1, & \text{in } a<t<2a \end{cases}$ with period $2a$. (N/D 2009)

5. Find the Laplace transform of square wave function (or Meoander function) of period

a as $f(t)=\begin{cases} 1, & \text{in } 0<t<\dfrac{a}{2} \\ -1, & \text{in } \dfrac{a}{2}<t<a \end{cases}$. (M/J 2013)

6. Find the Laplace transform of

$$\begin{aligned} f(t) &= \in, \quad 0\le t\le a \\ &= -\in, \quad a\le t\le 2a \end{aligned}$$

and $f(t+2a)=f(t)$ for all t. (N/D 2010)

7. Find the Laplace transform of a square wave function given by

$f(t)=\begin{cases} E & \text{for } 0\le t\le \dfrac{a}{2} \\ -E & \text{for } \dfrac{a}{2}\le t\le a \end{cases}$, and $f(t+a)=f(t)$. (N/D 2011)

8. Find the Laplace transform of the Half wave rectifier

$f(t)=\begin{cases} \sin\omega t, & 0<t<\pi/\omega \\ 0, & \pi/\omega<t<2\pi/\omega \end{cases}$ and $f(t+2\pi/\omega)=f(t)$ for all t.(N/D 2012)

- **Initial and Final Value Theorem& Other Simple Problems**

1. Find the Laplace transform of $te^{-2t}\cos 3t$. (M/J 2009)

2. Find $L\left[t^2e^{-3t}\sin 2t\right]$. (M/J 2013)

3. Verify initial and final value theorems for $f(t)=1+e^{-t}(\sin t+\cos t)$.

(M/J 2010),(N/D 2010),(M/J 2012)

4. Find $L\left[\dfrac{\cos at-\cos bt}{t}\right]$. (A/M 2011),(N/D 2012)

5. Find the Laplace transform of $f(t)=\dfrac{\sin^2 t}{t}$. (N/D 2014)

6. Find the Laplace transform of $\frac{e^{at}-e^{-bt}}{t}$. (M/J 2012)

7. Find the Laplace transform of $e^{-4t}\int_0^t t\sin 3t\,dt$. (M/J 2009)

8. Evaluate $\int_0^\infty te^{-2t}\cos t\,dt$ using Laplace transforms. (N/D 2011),(M/J 2012)

9. Find the value of $\int_0^\infty te^{-3t}\cos 2t\,dt$. (N/D 2014)

10. Find the inverse Laplace transform of $\frac{1}{(s+1)(s^2+4)}$. (M/J 2009)

11. Find the inverse Laplace transform of $\log\left(\frac{s+1}{s-1}\right)$. (N/D 2013)

12. Find $L^{-1}\left(\log\frac{s^2+1}{s(s+1)}\right)$. (N/D 2014)

13. Find $L^{-1}\left\{\frac{1}{s}\ln\left(\frac{s^2+a^2}{s^2+b^2}\right)\right\}$. (N/D 2011)(AUT)

14. Evaluate $L^{-1}\left(\frac{3s^2+16s+26}{s(s^2+4s+13)}\right)$. (N/D 2013)

- **Inverse Laplace Transform Using Convolution Theorem**

1. Using Convolution theorem $L^{-1}\left\{\frac{1}{(s+a)(s+b)}\right\}$. (A/M 2011)

2. Apply convolution theorem to evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$.
(M/J 2010),(M/J 2012),(N/D 2014)

3. Find $L^{-1}\left[\frac{s^2}{(s^2+4)^2}\right]$ using convolution theorem. (N/D 2012)

4. Find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ using convolution theorem.

(N/D 2010),(M/J 2011)

5. Using convolution theorem find the inverse Laplace transform of $\frac{1}{(s^2+1)(s+1)}$.

(N/D 2009),(N/D 2011)(AUT)

6. Find $L^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$ using convolution theorem. (N/D 2011)

7. Using convolution theorem find the inverse Laplace transform of $\frac{4}{(s^2+2s+5)^2}$.

(M/J 2013)

- **Solving Differential Equation By Laplace Transform**

1. Solve $\frac{d^2x}{dt^2}-3\frac{dx}{dt}+2x=2$, given $x=0$ and $\frac{dx}{dt}=5$ for $t=0$ using Laplace transform method. (A/M 2011),(N/D 2012)

2. Solve the equation $y''+9y=\cos 2t,\ y(0)=1$ and $y\left(\frac{\pi}{2}\right)=-1$ using Laplace transform. (M/J 2009),(N/D 2014)

3. Solve the differential equation $\frac{d^2y}{dt^2}+y=\sin 2t\ ;\ y(0)=0,\ y'(0)=0$ by using Laplace transform method. (N/D 2009)

4. Using Laplace transform solve the differential equation $y''-3y'-4y=2e^{-t}$ with $y(0)=1=y'(0)$. (M/J 2010),(N/D 2010)

5. Solve the differential equation $\frac{d^2y}{dt^2}-3\frac{dy}{dt}+2y=e^{-t}$ with $y(0)=1$ and $y'(0)=0$, using Laplace transform. (M/J 2012)

6. Solve $y''-3y'+2y=4e^{2t},\ y(0)=-3,\ y'(0)=5$, using Laplace transform.

(N/D 2011)(AUT)

7. Solve $y'' + 5y' + 6y = 2,\ y(0) = 0,\ y'(0) = 0$, using Laplace transform. (M/J 2013)

8. Solve, by Laplace transform method, the equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t$, $y(0) = 0, y'(0) = 1$. (M/J 2011)

9. Solve $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = \sin t$, if $\frac{dy}{dt} = 0$ and $y = 2$ when $t = 0$ using Laplace transforms. (N/D 2011)

10. Using Laplace transforms, solve $y'' + y' = t^2 + 2t,\ y(0) = 4,\ y'(0) = -2$. (N/D 2013)

----All the Best----