SUBJECT NAME : **Discrete Mathematics**

SUBJECT CODE : **MA 6566**

MATERIAL NAME : **University Questions**

REGULATION : **R2013**

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Unit – I (Logic and Proofs)

• Simplification by Truth Table and without Truth Table

1. Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent. (N/D 2012)

   Textbook Page No.: 1.3

2. Without using the truth table, prove that $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$.

   Textbook Page No.: 1.4 (N/D 2010)

3. Prove $\left((p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))\right) \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$ is a tautology.

   Textbook Page No.: 1.1.5 (N/D 2013),(A/M 2015),(A/M 2017)

4. Prove that $(P \rightarrow Q) \wedge (R \rightarrow Q) \Rightarrow (P \vee R) \rightarrow Q$. (M/J 2013)

   Textbook Page No.: 1.4

5. Show that $(\neg P \wedge (\neg Q \wedge R) \vee (Q \wedge R) \vee (P \wedge R)) \Leftrightarrow \mathbf{R}$, without using truth table.

   Textbook Page No.: 1.5 (A/M 2018)

6. Prove that $(p \rightarrow q) \wedge (q \rightarrow r) \Rightarrow (p \rightarrow r)$. (M/J 2014)

• PCNF and PDNF

1. Obtain the PDNF and PCNF of $(P \wedge Q) \vee (\neg P \wedge R)$. (N/D 2016)

   Textbook Page No.: 1.7

2. Find the PCNF of $(P \vee R) \wedge (P \vee \neg Q)$. Also find its PDNF, without using truth table.

   Textbook Page No.: 1.8 (A/M 2018)

3. Without using truth table find the PCNF and PDNF of
   $P \rightarrow (Q \wedge P) \wedge (\neg P \rightarrow (\neg Q \wedge \neg R))$. (A/M 2011)

   Textbook Page No.: 1.9

4. Find the principal disjunctive normal form of the statement,
   $(q \vee (p \wedge r)) \wedge \sim ((p \vee r) \wedge q)$. (N/D 2012)

   Textbook Page No.: 1.10

5. Obtain the principal disjunctive normal form and principal conjunction form of the statement $p \vee (\neg p \rightarrow (q \vee (\neg q \rightarrow r)))$. (N/D 2010)

   Textbook Page No.: 1.11

6. Obtain the principal conjunctive normal form and principal disjunctive normal form of $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$ by using equivalences. (M/J 2016),(A/M 2017)

   Textbook Page No.: 1.12

7. Show that $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P) = (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge$
   $(P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$. (M/J 2013)

   Textbook Page No.: 1.13

- ## Theory of Inference

1. Show that: $(P \rightarrow Q) \wedge (R \rightarrow S),\ (Q \wedge M) \wedge (S \rightarrow N),\ \neg(M \wedge N)$ and
   $(P \rightarrow R) \Rightarrow \neg P$. (A/M 2011)

   Textbook Page No.: 1.15

2. Show that $(p \rightarrow q) \wedge (r \rightarrow s),\ (q \rightarrow t) \wedge (s \rightarrow u),\ \neg(t \wedge u)$ and
   $(p \rightarrow r) \Rightarrow \neg p$. (A/M 2015)

   Textbook Page No.: 1.15

3. Prove that the following argument is valid: $p \to \neg q,\ r \to q,\ r \Rightarrow \neg p$.

   Textbook Page No.: 1.14 (M/J 2012)

4. Prove that the premises $P \to Q$, $Q \to R$, $R \to S$, $S \to \sim R$ and $P \wedge S$ are inconsistent. (N/D 2014)

   Textbook Page No.: 1.16

5. Show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\neg R \vee P$ and Q. (N/D 2015),(M/J 2016),(A/M 2017)

   Textbook Page No.: 1.17

6. Show that using rule C.P $\neg P \vee Q,\ \neg Q \vee R,\ R \to S \Rightarrow P \to S$. (A/M 2018)

   Textbook Page No.: 1.18

7. Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \to R$, $P \to M$, $\neg M$. (N/D 2016)

   Textbook Page No.: 1.18

8. Prove that the premises $a \to (b \to c),\ d \to (b \wedge \neg c)$ and $(a \wedge d)$ are inconsistent.

   Textbook Page No.: 1.19 (N/D 2010)

9. Using indirect method of proof, derive $p \to \neg s$ from the premises $p \to (q \vee r), q \to \neg p, s \to \neg r$ and p. (N/D 2011)

   Textbook Page No.: 1.20

10. Prove that $A \to \neg D$ is a conclusion from the premises $A \to B \vee C$, $B \to \neg A$ and $D \to \neg C$ by using conditional proof. (M/J 2014)

    Textbook Page No.: 1.21

11. Show that the hypothesis, “It is not sunny this afternoon and it is colder than yesterday”, “we will go swimming only if it is sunny”, “If we do not go swimming, then we will take a canoe trip” and “If we take a canoe trip, then we will be home by sunset” lead to the conclusion “We will be home by sunset”. (N/D 2012),(N/D 2013)

    Textbook Page No.: 1.24

12. Show that "It rained" is a conclusion obtained from the statements.

"If it does not rain or if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on". "If the sports day is held, the trophy will be awarded" and "the trophy was not awarded". (M/J 2016)

13. Determine the validity of the following argument:

If 7 is less than 4, then 7 is not a prime number, 7 is not less than 4. Therefore 7 is a prime number. (M/J 2012)

Textbook Page No.: 1.25

14. Prove that $\sqrt{2}$ is irrational by giving a proof using contradiction.

Textbook Page No.: 1.22 (N/D 2011),(M/J 2013),(N/D 2013),(M/J 2016)

- **Quantifiers**

1. Show that $(\forall x)(P(x) \rightarrow Q(x)),\ (\exists y)P(y) \Rightarrow (\exists x)Q(x)$. (M/J 2012)

Textbook Page No.: 1.26

2. Use the indirect method to prove that the conclusion $\exists z Q(z)$ follows form the premises $\forall x(P(x) \rightarrow Q(x))$ and $\exists y P(y)$. (N/D 2012)

Textbook Page No.: 1.27

3. Show that $(x)[P(x) \rightarrow Q(x)] \wedge (x)[Q(x) \rightarrow R(x)] \Rightarrow (x)[P(x) \rightarrow R(x)]$.

Textbook Page No.: 1.27 (N/D 2016)

4. Prove that $\forall x(P(x) \rightarrow Q(x)),\ \forall x(R(x) \rightarrow \neg Q(x)) \Rightarrow \forall x(R(x) \rightarrow \neg P(x))$.

Textbook Page No.: 1.28 (N/D 2010)

5. Use indirect method of proof to prove that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$. (A/M 2011),(N/D 2011),(A/M 2015)

Textbook Page No.: 1.29

6. Show that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$. Is the converse true?

Textbook Page No.: 1.30 (N/D 2013)

7. Show that $(\exists x)P(x) \to \forall x Q(x) \Rightarrow (x)(P(x) \to Q(x))$. (M/J 2014)

8. Show that the premises "One student in this class knows how to write programs in JAVA" and "Everyone who knows how to write programs in Java can get a high paying job imply a conclusion "Someone in this class can get a high paying job". (N/D 2015)

   Textbook Page No.: 1.31

9. Use rules of inferences to obtain the conclusion of the following arguments:

   "Babu is a student in this class, knows how to write programmes in JAVA". "Everyone who knows how to write programmes in JAVA can get a high-paying job". Therefore, "someone in this class can get a high-paying job". (A/M 2017)

   Textbook Page No.: 1.31

10. Show that the statement "Every positive integer is the sum of the squares of three integers" is false. (N/D 2011)

    Textbook Page No.: 1.32

11. Verify the validity of the following argument. Every living thing is a plant or an animal. John's gold fish is alive and it is not a plant. All animals have hearts. Therefore John's gold fish has a heart. (M/J 2012)

    Textbook Page No.: 1.32

12. Write the symbolic form and negate the following statements: (A/M 2015)

    (i) Every one who is healthy can do all kinds of work.

    (ii) Some people are not admired by every one.

    (iii) Every one should help his neighbors, or his neighbors will not help him.

    (iv) Every one agrees with some one and some one agrees with every one.

13. Verify that validating of the following inference.

    If one person is more successful than another, then he has worked harder to deserve success. Ram has not worked harder than Siva. Therefore, Ram is not more successful than Siva. (A/M 2011)

    Textbook Page No.: 1.34

Unit – II (Combinatorics)

• Mathematical Induction and Strong Induction

1. Prove by the principle of mathematical induction, for $'n'$ a positive integer,

   $1^2+2^2+3^2+...+n^2=\frac{n(n+1)(2n+1)}{6}$. (M/J 2012),(A/M 2015)

   Textbook Page No.: 2.1

2. Use Mathematical induction show that $\sum_{k=1}^{n} k^2=\frac{n(n+1)(2n+1)}{6}$. (A/M 2011)

   Textbook Page No.: 2.1

3. Using mathematical induction show that $\sum_{r=1}^{n} 3^r=\frac{3^{n+1}-1}{2}$. (M/J 2016),(A/M 2017)

   Textbook Page No.: 2.3

4. Using mathematical induction to show that $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{n}}>\sqrt{n},\ n\geq 2$.

   Textbook Page No.: 2.4 (N/D 2011),(N/D 2016)

5. Prove by mathematical induction that $6^{n+2}+7^{2n+1}$ is divisible by 43 for each positive integer n. (N/D 2013)

   Textbook Page No.: 2.5

6. Prove, by mathematical induction, that for all $n\geq 1,\ n^3+2n$ is a multiple of 3.

   Textbook Page No.: 2.7 (N/D 2010),(N/D 2015)

7. Use Mathematical induction to prove the inequality $n<2^n$ for all positive integer n.

   Textbook Page No.: 2.8 (N/D 2012)

8. Prove that the number of subsets of set having n elements is 2^n. (M/J 2014)

   Textbook Page No.: 2.9

9. State the Strong Induction (the second principle of mathematical induction). Prove that a positive integer greater than 1 is either a prime number or it can be written as product of prime numbers. (M/J 2013)

   Textbook Page No.: 2.10

• Pigeonhole Principle

1. Let m any odd positive integer. Then prove that there exists a positive integer n such that m divides $2^n - 1$. (M/J 2013)

   Textbook Page No.: 2.12

2. Prove that in a group of six people, atleast three must be mutual friends or atleast three must be mutual strangers. (N/D 2015)

   Textbook Page No.: 2.13

3. If n Pigeonholes are occupied by $(kn+1)$ pigeons, where k is positive integer, prove that at least one Pigeonhole is occupied by $k+1$ or more Pigeons. Hence, find the minimum number of m integers to be selected from $S = \{1, 2, ..., 9\}$ so that the sum of two of the m integers are even. (N/D 2011)

   Textbook Page No.: 2.14

4. What is the maximum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade if there are five possible grades A, B, C, D and E? (N/D 2012)

   Textbook Page No.: 2.15

• Permutations and Combinations

1. How many positive integers n can be formed using the digits 3, 4, 4, 5, 5, 6, 7 if n has to exceed 5000000? (N/D 2010)

   Textbook Page No.: 2.16

2. Find the number of distinct permutations that can be formed from all the letters of each word (1) RADAR (2) UNUSUAL. (M/J 2012)

   Textbook Page No.: 2.17

3. From a club consisting of six men and seven women, in how many ways we select a committee of (1) 3 men and four women? (2) 4 person which has at least one women? (3) 4 person that has at most one man? (4) 4 persons that has children of both sexes?

(N/D 2015)

4. There are six men and five women in a room. Find the number of ways four persons can be drawn from the room if (1) they can be male or female, (2) two must be men and two women, (3) they must all are of the same sex. (M/J 2016),(A/M 2017)

Textbook Page No.:

5. Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members form the mathematics department and four from the computer science department? (N/D 2012)

Textbook Page No.: 2.20

6. A box contains six white balls and five red balls. Find the number of ways four balls can be drawn from the box if (1) They can be any colour (2) Two must be white and two red (3) They must all be the same colour. (A/M 2011)

Textbook Page No.: 2.19

• Solving recurrence relations by generating function

1. A factory makes custom sports cars at an increasing rate. In the first month only one car is made, in the second month two cars are made and so on, with n cars made in the n^{th} month. (N/D 2013)

(i) Set up recurrence relation for the number of cars produced in the first n months by this factory.

(ii) How many cars are produced in the first year?

Textbook Page No.: 2.22

2. Solve the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ given that $a_0 = 5,\ a_1 = 9$ and $a_2 = 15$. (M/J 2014)

Textbook Page No.: 2.23

3. Solve the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with $a_0 = 5,\ a_1 = -9$ and $a_2 = 15$. (N/D 2014)

   Textbook Page No.: 2.24

4. Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$, with the initial conditions $a_0 = 2,\ a_1 = 5$ and $a_2 = 15$. (N/D 2014)

   Textbook Page No.: 2.24

5. Find the generating function of Fibonacci sequence. (N/D 2013)

   Textbook Page No.: 2.26

6. Solve using generating function: $S(n) + 3S(n-1) - 4S(n-2) = 0$; $n \geq 2$ given $S(0) = 3$, $S(1) = -2$. (A/M 2018)

   Textbook Page No.: 2.27

7. Using generating function, solve the recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} = 0$ where $n \geq 2,\ a_0 = 0$ and $a_1 = 1$. (M/J 2013)

   Textbook Page No.: 2.29

8. Using the generating function, solve the difference equation $y_{n+2} - y_{n+1} - 6y_n = 0,\ y_1 = 1,\ y_0 = 2$. (N/D 2010)

   Textbook Page No.: 2.31

9. Using generating function solve $y_{n+2} - 5y_{n+1} + 6y_n = 0,\ n \geq 0$ with $y_0 = 1$ and $y_1 = 1$.

   Textbook Page No.: 2.31 (A/M 2011)

10. Solve the recurrence relation $a_n - 7a_{n-1} + 6a_{n-2} = 0$, for $n \geq 2$ with initial conditions $a_0 = 8$ and $a_1 = 6$, using generating function. (A/M 2017)

    Textbook Page No.: 2.33

11. Use generating functions to solve the recurrence relation $a_n + 3a_{n-1} - 4a_{n-2} = 0,\ n \geq 2$ with the initial condition $a_0 = 3,\ a_1 = -2$.

    Textbook Page No.: 2.35 (N/D 2012),(N/D 2015)

12. Solve the recurrence relation $a_n = 3a_{n-1} + 2, \ \ n \geq 1$, with $a_0 = 1$ by the method of generating functions. (M/J 2014)

    Textbook Page No.: 2.36

13. Use the method of generating function to solve the recurrence relation $a_n = 3a_{n-1} + 1, \ \ n \geq 1$ given that $a_0 = 1$. (A/M 2015)

    Textbook Page No.: 2.38

14. Use generating function to solve the recurrence relation $S(n+1) - 2S(n) = 4^n$ with $S(0) = 1$, $n \geq 0$. (M/J 2016)

    Textbook Page No.: 2.39

15. Using method of generating function to solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 4^n; \ \ n \geq 2$, given that $a_0 = 2$ and $a_1 = 8$. (N/D 2011)

    Textbook Page No.: 2.40

16. Solve the recurrence relation $a_{n+1} - a_n = 3n^2 - n, \ \ n \geq 0, \ a_0 = 3$. (N/D 2011)

- ## Inclusion and Exclusion

1. Find the number of integers between 1 and 250 both inclusive that are divisible by any of the integers 2, 3, 5, 7. (N/D 2010),(M/J 2016)

   Textbook Page No.: 2.43

2. Find the number of integers between 1 and 250 both inclusive that are not divisible by any of the integers 2, 3, 5 and 7. (A/M 2015),(N/D 2016),(A/M 2018)

   Textbook Page No.: 2.44

3. Find the number of integers between 1 and 500 that are not divisible by any of the integers 2, 3, 5 and 7. (A/M 2017)

   Textbook Page No.: 2.45

4. Determine the number of positive integers $n, \ \ 1 \leq n \leq 2000$ that are not divisible by 2, 3 or 5 but are divisible by 7. (M/J 2013)

   Textbook Page No.: 2.48

5. Find the number of positive integers ≤ 1000 and not divisible by any of $3, 5, 7$ and 22. (M/J 2014)

Textbook Page No.: 2.46

6. There are 2500 students in a college, of these 1700 have taken a course in C, 1000 have taken a course Pascal and 550 have taken a course in Networking. Further 750 have taken courses in both C and Pascal. 400 have taken courses in both C and Networking, and 275 have taken courses in both Pascal and Networking. If 200 of these students have taken courses in C, Pascal and Networking.

   (1) How many of these 2500 students have taken a course in any of these three courses C, Pascal and Networking?

   (2) How many of these 2500 students have not taken a course in any of these three courses C, Pascal and Networking? (A/M 2011)

Textbook Page No.: 2.49

7. A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken atleast one of Spanish, French and Russian, how many students have taken a course in all three languages? (N/D 2013)

Textbook Page No.: 2.50

Unit – III (Graphs)

• Drawing graphs from given conditions

1. Draw the complete graph K_5 with vertices A, B, C, D, E. Draw all complete sub graph of K_5 with 4 vertices. (N/D 2010)

Textbook Page No.: 3.3

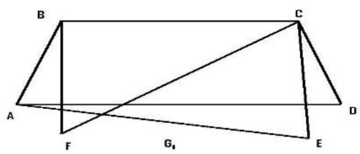
2. Draw the graph with 5 vertices, A, B, C, D, E such that $\deg(A) = 3$, B is an odd vertex, $\deg(C) = 2$ and D and E are adjacent. (N/D 2010)

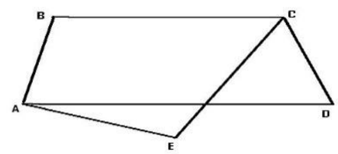
Textbook Page No.: 3.4

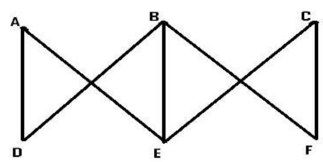
3. Draw the graph with 5 vertices A, B, C, D and E such that $\deg(A) = 3$, B is an odd vertex, $\deg(C) = 2$ and D and E are adjacent. (A/M 2011)

Textbook Page No.: 3.4

4. Determine which of the following graphs are bipartite and which are not. If a graph is bipartite, state if it is completely bipartite. (N/D 2011)



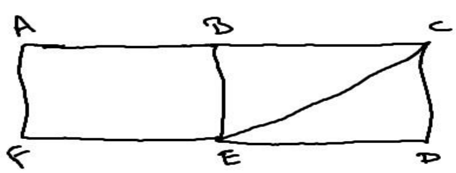




Textbook Page No.: 3.5

5. Find the all the connected sub graph obtained from the graph given in the following Figure, by deleting each vertex. List out the simple paths from A to in each sub graph.

Textbook Page No.: 3.7 (A/M 2011)



6. Given an example of a graph which is (N/D 2016)

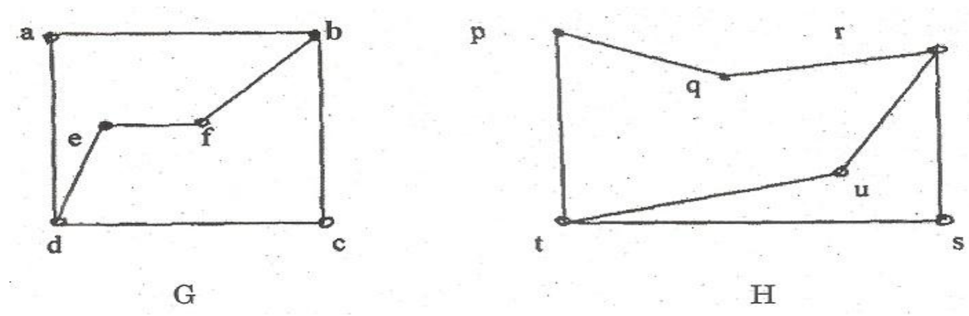
(i) Euleran but not Hamiltonian (ii) Hamiltonian but not Eulerian

(iii) Hamiltonian and Eulerian (iv) Neither Hamiltonian nor Eulerian

Textbook Page No.: 3.9

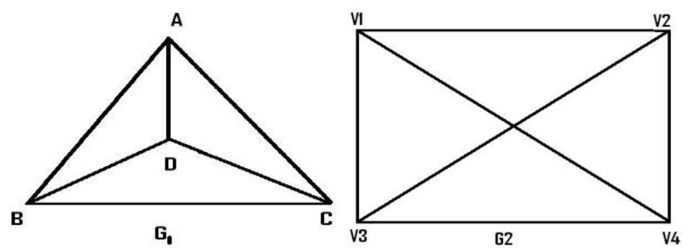
• Isomorphism of graphs

1. Determine whether the graphs G and H given below are isomorphic. (N/D 2012)



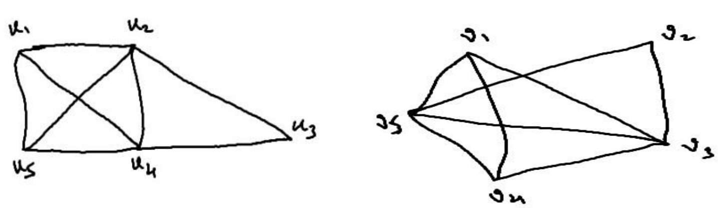
Textbook Page No.: 3.11

2. Using circuits, examine whether the following pairs of graphs G_1, G_2 given below are isomorphic or not: (N/D 2011),(N/D 2016)



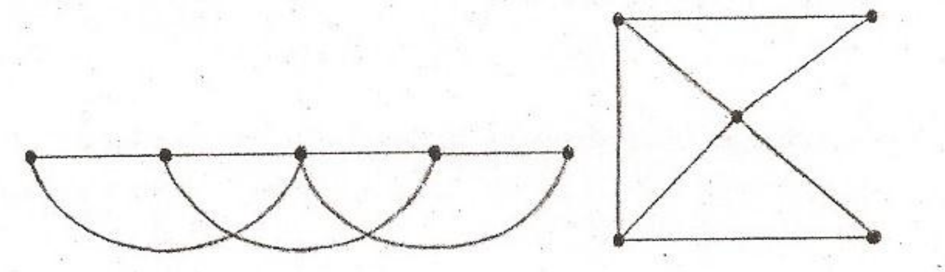
Textbook Page No.: 3.12

3. Examine whether the following pair of graphs are isomorphic. If not isomorphic, give the reasons. (A/M 2011)



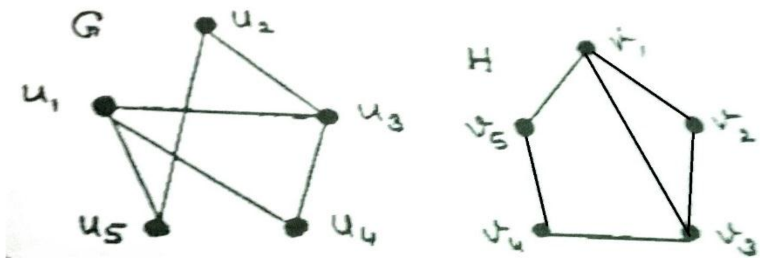
Textbook Page No.: 3.14

4. Check whether the two graphs given are isomorphic or not. (M/J 2013)



Textbook Page No.: 3.15

5. Determine whether the following graphs G and H are isomorphic. (N/D 2013)



Textbook Page No.: 3.16

6. Define isomorphism between two graphs. Are the simple graphs with the following adjacency matrices isomorphic? (M/J 2016),(A/M 2017)

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

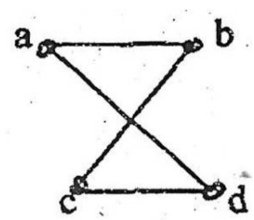
Textbook Page No.: 3.19

7. The adjacency matrices of two pairs of graph as given below. Examine the isomorphism of G and H by finding a permutation matrix. $A_G = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, A_H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

Textbook Page No.: 3.20 (N/D 2010)

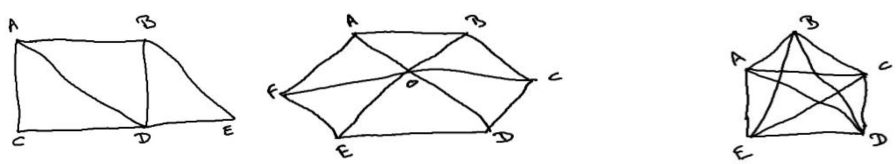
• General problems in graphs

1. How many paths of length four are there from a and d in the simple graph G given below. (N/D 2012)



Textbook Page No.: 3.21

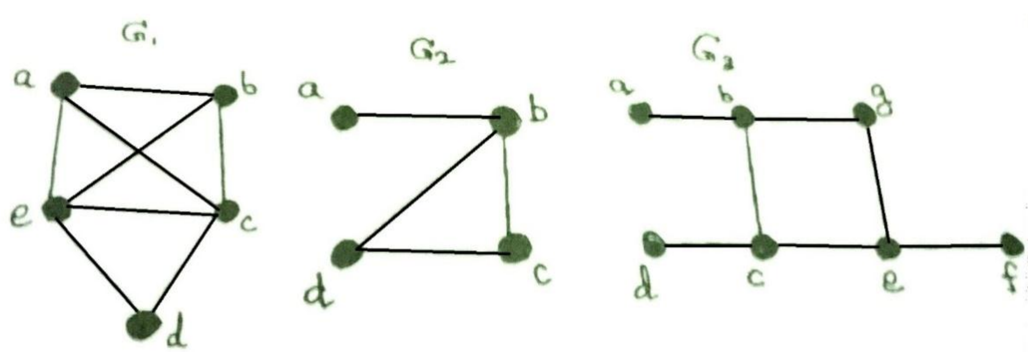
2. Find an Euler path or an Euler circuit, if it exists in each of the three graphs below. If it does not exist, explain why? (N/D 2011),(A/M 2015)



Textbook Page No.: 3.22

3. Which of the following simple graphs have a Hamilton circuit or, if not, a Hamilton path?

Textbook Page No.: 3.23



• Theorems

1. Prove that an undirected graph has an even number of vertices of odd degree.

Textbook Page No.: 3.24 (N/D 2012),(M/J 2014)

2. Prove that the number of vertices of odd degree in any graph is even.

   Textbook Page No.: 3.24 (A/M 2015),(N/D 2015),(M/J 2016),(A/M 2017)

3. State and prove hand shaking theorem. Also prove that maximum number of edges in a connected graph with n vertices is $\frac{n(n-1)}{2}$. (N/D 2016),(A/M 2018)

   Textbook Page No.: 3.25

4. Prove that the complement of a disconnected graph is connected. (A/M 2017)

   Textbook Page No.: 3.27

5. Show that if a graph with n vertices is self-complementary then $n \equiv 0 \text{ or } 1 \pmod 4$.

   Textbook Page No.: 3.28 (M/J 2013),(M/J 2016)

6. Prove that the maximum number of edges in a simple disconnected graph G with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$. (N/D 2011),(A/M 2015),(N/D 2015)

   Textbook Page No.: 3.29

7. Prove that a simple graph with n vertices and k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges. (N/D 2013)

   Textbook Page No.: 3.29

8. Let G be a graph with exactly two vertices has odd degree. Then prove that there is a path between those two vertices. (N/D 2016)

   Textbook Page No.: 3.31

9. Show that graph G is disconnected if and only if its vertex set V can be partitioned into two nonempty subsets V_1 and V_2 such that there exists no edge is G whose one end vertex is in V_1 and the other in V_2. (M/J 2012),(A/M 2018)

   Textbook Page No.: 3.32

10. If all the vertices of an undirected graph are each of degree k, show that the number of edges of the graph is a multiple of k. (N/D 2010)

    Textbook Page No.: 3.33

11. Let G be a simple undirected graph with adjacency matrix A with respect to the ordering $v_1, v_2, v_3, ..., v_n$. Prove that the number of different walks of length r from v_i to v_j equals the (i, j) th entry of A^r, where r is a positive integer. (M/J 2013)

    Textbook Page No.: 3.34

• Theorems based on Euler and Hamilton graph

1. Prove that a connected graph G is Eulerian if and only if all the vertices are of even degree. (M/J 2012),(N/D 2013),(M/J 2014),(N/D 2015),(A/M 2018)

   Textbook Page No.: 3.36

2. Show that the complete graph with n vertices K_n has a Hamiltonian circuit whenever $n \geq 3$. (N/D 2012)

   Textbook Page No.: 3.37

3. Prove that if G is a simple graph with at least three vertices and $\delta(\mathrm{G}) \geq \frac{|V(G)|}{2}$ then G is Hamiltonian. (M/J 2013)

   Textbook Page No.: 3.37

4. Let G be a simple undirected graph with n vertices. Let u and v be two non adjacent vertices in G such that $\deg(u)+\deg(v) \geq n$ in G. Show that G is Hamiltonian if and only if $G+uv$ is Hamiltonian. (A/M 2011)

   Textbook Page No.: 3.40

5. If G is a connected simple graph with n vertices with $n \geq 3$, such that the degree of every vertex in G is at least $\frac{n}{2}$, then prove that G has Hamilton cycle.

   Textbook Page No.: 3.38 (M/J 2016),(A/M 2017)

Unit – IV (Algebraic Structures)

• Group, Subgroup and Normal Subgroup

1. Prove that $G=\left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix},\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix},\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}\right\}$ forms an abelian group under matrix multiplication. (N/D 2015)

   Textbook Page No.: 4.2

2. Show that M_2, the set of all 2×2 non-singular matrices over R is a group under usual matrix multiplication. Is it abelian? (A/M 2015)

   Textbook Page No.: 4.3

3. If $(G,*)$ is an abelian group, show that $(a*b)^2 = a^2 * b^2$. (N/D 2010)

   Textbook Page No.: 4.4

4. In any group $\langle G,*\rangle$, show that $(a*b)^{-1} = b^{-1} * a^{-1}$, for all $a,b \in G$. (M/J 2016)

   Textbook Page No.: 4.5

5. Show that $(Q,*)$ is an abelian group, where $*$ is defined by $a*b=\dfrac{ab}{2}, \forall a,b\in Q^+$.

   Textbook Page No.: 4.6 (N/D 2016),(A/M 2018)

6. If * is a binary operation on the set R of real numbers defined by $a*b=a+b+2ab$,

   (1) Find $\langle R,*\rangle$ is a semi group

   (2) Find the identity element if it exists

   (3) Which elements has inverse and what are they? (A/M 2011)

   Textbook Page No.: 4.8

7. If $S=N\times N$, the set of ordered pairs of positive integers with the operation $*$ defined by $(a,b)*(c,d)=(ad+bc,bd)$ and if $f:(S,*)\to(Q,+)$ is defined by $f(a,b)=\dfrac{a}{b}$, show that f is a semigroup homomorphism. (A/M 2015)

Textbook Page No.: 4.11

8. Find the left cosets of the subgroup $H = \{[0],[3]\}$ of the group $[z_6, +_6]$.

Textbook Page No.: 4.12 (M/J 2014)

9. State and prove Lagrange's theorem. (N/D 2010),(A/M 2011),(M/J 2012),(N/D 2013),(M/J 2014),(A/M 2015),(M/J 2016),(N/D 2016),(A/M 2017)

Textbook Page No.: 4.13

10. Prove that the order of a subgroup of a finite group divides the order of the group.

Textbook Page No.: 4.13 (N/D 2011),(M/J 2013),(A/M 2018)

11. Find all the subgroups of $(z_9, +_9)$. (M/J 2014)

Textbook Page No.: 4.15

12. Prove the theorem: Let $\langle G, * \rangle$ be a finite cyclic group generated by an element $a \in G$. If G is of order n, that is, $|G| = n$, then $a^n = e$, so that $G = \{a, a^2, a^3, \ldots, a^n = e\}$. Further more n is a least positive integer for which $a^n = e$. (N/D 2011)

Textbook Page No.: 4.16

13. Prove that the intersection of two subgroups of a group G is again a subgroup of G.

Textbook Page No.: 4.16 (N/D 2015)

14. Prove that intersection of any two subgroups of a group $(G, *)$ is again a subgroup of $(G, *)$. (N/D 2013)

Textbook Page No.: 4.16

15. Prove that intersection of two normal subgroups of a group $(G, *)$ is a normal subgroup of a group $(G, *)$. (M/J 2013),(N/D 2016),(A/M 2018)

Textbook Page No.: 4.18

16. Show that the union of two subgroups of a group G is a subgroup of G if and only if one is contained in the other. (A/M 2015)

17. Prove that every cyclic group is an abelian group. (N/D 2013)

Textbook Page No.: 4.18

18. Prove that every subgroup of a cyclic group is cyclic. (M/J 2016),(A/M 2017)

Textbook Page No.: 4.19

19. Prove that the necessary and sufficient condition for a non empty subset H of a group $\{G,*\}$ to be a sub group is $a,b \in H \Rightarrow a*b^{-1} \in H$. (N/D 2012)

Textbook Page No.: 4.20

20. If $*$ is the operation defined on $S = Q \times Q$, the set of ordered pairs of rational numbers and given by $(a,b)*(x,y) = (ax, ay+b)$, show that $(S,*)$ is a semi group. Is it commutative? Also find the identity element of S. (N/D 2012)

Textbook Page No.: 4.9

21. Define the Dihedral group $\langle D_4,* \rangle$ and give its composition table. Hence find the identify element and inverse of each element. (A/M 2011)

Textbook Page No.: 4.22

- ## Homomorphism and Isomorphism

1. Prove that the group homomorphism preserves the identity element. (N/D 2015)

Textbook Page No.: 4.24

2. Prove that every finite group of order n is isomorphic to a permutation group of order n. (N/D 2011),(M/J 2013)

Textbook Page No.: 4.24

3. State and prove the fundamental theorem of group homomorphism.

Textbook Page No.: 4.29 (N/D 2013)

4. Let $f : G \to G'$ be a homomorphism of groups with Kernel K. Then prove that K is a normal subgroup of G and G/K is isomorphic to the image of f. (M/J 2012)

Textbook Page No.: 4.29

5. Let $(G,*)$ and (H,Δ) be two groups and $g:(G,*) \to (H,\Delta)$ be group homomorphism. Prove that the Kernel of g is normal subgroup of $(G,*)$.

Textbook Page No.: 4.27 (M/J 2013),(M/J 2016),(A/M 2017)

6. Show that the Kernel of a homomorphism of a group $\langle G,*\rangle$ into an another group $\langle H,\Delta\rangle$ is a subgroup of G . (A/M 2011),(A/M 2018)

   Textbook Page No.: 4.29

7. If $f:G\to G'$ is a group homomorphism from $\{G,*\}$ to $\{G',\Delta\}$ then prove that for any $a\in G,\ f\left(a^{-1}\right)=\left[f(a)\right]^{-1}$. (N/D 2012)

   Textbook Page No.: 4.31

8. If $(Z,+)$ and $(E,+)$ where Z is the set all integers and E is the set all even integers, show that the two semi groups $(Z,+)$ and $(E,+)$ are isomorphic. (N/D 2010)

   Textbook Page No.: 4.32

9. Let $(S,*)$ be a semi group. Then prove that there exists a homomorphism $g:S\to S^S$, where $\langle S^S,\circ\rangle$ is a semi group of functions from S to S under the operation of (left) composition. (N/D 2011)

   Textbook Page No.: 4.33

- **Ring and Fields**

1. Show that $(Z,+,\times)$ is an integral domain where Z is the set of all integers.

   Textbook Page No.: 4.34 (N/D 2010)

2. Prove that the set $Z_4=\{[0],[1],[2],[3]\}$ is a commutative ring with respect to the binary operation addition modulo and multiplication modulo $+_4$ and $\times_4$.

   Textbook Page No.: 4.35 (N/D 2012),(N/D 2015)

Unit – V (Lattices and Boolean algebra)

• Partially Ordered Set (Poset)

1. Show that $(N, \leq)$ is a partially ordered set where N is set of all positive integers and $\leq$ is defined by $m \leq n$ iff $n - m$ is a non-negative integer. (N/D 2010),(A/M 2018)

   Textbook Page No.: 5.2

2. Draw the Hasse diagram for (1) $P_1 = \{2,3,6,12,24\}$ (2) $P_2 = \{1,2,3,4,6,12\}$ and $\leq$ is a relation such $x \leq y$ if and only is $x \mid y$. (A/M 2011)

   Textbook Page No.: 5.4

3. Draw the Hasse diagram representing the partial ordering $\{(A,B): A \subseteq B\}$ on the power set $P(S)$ where $S = \{a,b,c\}$. Find the maximal, minimal, greatest and least elements of the poset. (N/D 2012)

   Textbook Page No.: 5.5

4. Consider the Lattice D_{105} with partial ordered relation divides, then (N/D 2016)

   (i) Draw the Hasse diagram of D_{105} (ii) Find the complement of each elements of D_{105}
   (iii) Find the set of atoms of D_{105} (iv) Find the number of sub algebras of D_{105}

   Textbook Page No.: 5.35

5. Let D_{30} with D if and only if x divides y. Find the following (A/M 2018)

   i) All lower bounds of 10 and 15 Textbook Page No.: 5.6

   ii) GLB of 10 and 15

   iii) All upper bounds of 10 and 15

   iv) LUB of 10 and 15

   v) Draw the Hasse diagram for D_{30}

• Lattices

1. In a distributive lattice prove that $a*b=a*c$ and $a\oplus b=a\oplus c$ imply $b=c$.

   Textbook Page No.: 5.12 (M/J 2014),(A/M 2018)

2. Let L be lattice, where $a*b=\text{glb}(a,b)$ and $a\oplus b=\text{lub}(a,b)$ for all $a,b\in L$. Then both binary operations * and $\oplus$ defined as in L satisfies commutative law, associative law, absorption law and idempotent law. (M/J 2013)

   Textbook Page No.: 5.13

3. Show that every ordered lattice $\{L,\vee,\wedge\}$ satisfies the following properties of the algebraic lattice (i) idempotent (ii) commutative (iii) associative (iv) absorption.

   Textbook Page No.: 5.13 (A/M 2017)

4. In a distributive Lattice $\{L,\vee,\wedge\}$ if an element $a\in L$ a complement then it is unique.

   Textbook Page No.: 5.13 (N/D 2012),(N/D 2016),(A/M 2018)

5. Show that every chain is a lattice. (M/J 2013)

   Textbook Page No.: 5.16

6. Prove that every chain is modular. (M/J 2016)

   Textbook Page No.: 5.18

7. Prove that every chain is a distributive lattice.

   Textbook Page No.: 5.16 (N/D 2013),(A/M 2015),(M/J 2016),(N/D 2016),(A/M 2017)

8. Prove that every distributive lattice is modular. Is the converse true? Justify your claim.

   Textbook Page No.: 5.18 (A/M 2011)

9. Show that in a distributive and complemented lattice $a\le b \Leftrightarrow a*b'=0 \Leftrightarrow a'\oplus b=1 \Leftrightarrow b'\le a'$. (M/J 2013)

   Textbook Page No.: 5.19

10. In a distributive complemented lattice. Show that the following are equivalent.

    (i) $a\le b$ (ii) $a\wedge\overline{b}=0$ (iii) $\overline{a}\vee b=1$ (iv) $\overline{b}\le\overline{a}$ (M/J 2016),(N/D 2016),(A/M 2017)

    Textbook Page No.: 5.19

11. Show that in a lattice if $a \le b \le c$, then (N/D 2013)

    (i) $a \oplus b = b * c$

    (ii) $(a*b) \oplus (b*c) = b = (a \oplus b) * (a \oplus c)$

    Textbook Page No.: 5.21

12. Show that the direct product of any two distributive lattices is a distributive lattice.

    Textbook Page No.: 5.22 (A/M 2011) ,(M/J 2012)

13. If $P(S)$ is the power set of a set S and $\cup, \cap$ are taken as join and meet, prove that $\langle P(S), \subseteq \rangle$ is a lattice. Also, prove the modular inequality of a Lattice $\langle L, \le \rangle$ for any $a,b,c \in L$, $a \le c \Leftrightarrow a \vee (b \wedge c) \le (a \vee b) \wedge c$. (N/D 2011)

    Textbook Page No.: 5.24

14. Prove that Demorgan's laws hold good for a complemented distributive lattice $\langle L, \wedge, \vee \rangle$, viz $(a \vee b)' = a' \wedge b'$ and $(a \wedge b)' = a' \vee b'$. (N/D 2011),(M/J 2013)

    Textbook Page No.: 5.25

15. State and prove De Morgan's laws in a complemented, distributive lattice.

    Textbook Page No.: 5.25 (M/J 2014),(A/M 2015)

16. If S_{42} is the set all divisors of 42 and D is the relation "divisor of" on S_{42}, prove that $\{S_{42}, D\}$ is a complemented Lattice. (N/D 2010)

    Textbook Page No.: 5.9

17. If S_n is the set of all divisors of the positive integer n and D is the relation of 'division', prove that $\{S_{30}, D\}$ is a lattice. Find also all the sub lattices of $\{S_{30}, D\}$ that contains 6 or more elements. (A/M 2015)

    Textbook Page No.: 5.7

• Boolean Algebra

1. In a Boolean algebra, prove that $(a \wedge b)' = a' \vee b'$. (N/D 2010)

   Textbook Page No.: 5.27

2. Prove that in a Boolean algebra $(a \vee b)' = a' \wedge b'$. (N/D 2015)

   Textbook Page No.: 5.27

3. Show that the De Morgan's laws hold in a Boolean algebra. (N/D 2014),(M/J 2016)

   Textbook Page No.: 5.27

4. In any Boolean algebra, show that $ab' + a'b = 0$ if and only if $a = b$. (N/D 2011)

   Textbook Page No.: 5.28

5. In any Boolean algebra, prove that the following statements are equivalent:

   (1) $a + b = b$ (2) $a \bullet b = a$ (3) $a' + b = 1$ and (4) $a \bullet b' = 0$ (N/D 2011)

   Textbook Page No.: 5.29

6. In a Boolean algebra, prove that $a.(a+b) = a$, for all $a, b \in B$. (N/D 2012)

   Textbook Page No.: 5.30

7. Simplify the Boolean expression $a'.b'.c + a.b'.c + a'.b'.c'$ using Boolean algebra identities. (N/D 2012)

   Textbook Page No.: 5.31

8. In any Boolean algebra, show that $(a+b')(b+c')(c+a') = (a'+b)(b'+c)(c'+a)$.

   Textbook Page No.: 5.32 (N/D 2013),(M/J 2014)

9. Let B be a finite Boolean algebra and let A be the set of all atoms of B. Then prove that the Boolean algebra B is isomorphic to the Boolean algebra $P(A)$, where $P(A)$ is the power set of A. (M/J 2012)

   Textbook Page No.: 5.33

10. If $P(S)$ is the power set of a non-empty S, prove that $\{P(S),\cup,\cap,\setminus,\phi,S\}$ is a Boolean algebra. (N/D 2015)

11. Prove that D_{110}, the set of all positive divisors of a positive integer 110, is a Boolean algebra and find all its sub algebras. (A/M 2011)

    Textbook Page No.: 5.37

12. If $a,b \in S\{1,2,3,6\}$ and $a+b = LCM(a,b)$, $a \bullet b = GCD(a,b)$ and $a' = \frac{6}{a}$, show that $\{S,+,\bullet,',1,6\}$ is a Boolean algebra. (A/M 2015)

    Textbook Page No.: 5.39

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