SUBJECT NAME : **Numerical Methods**

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TEXTBOOK FOR REFERENCE : **Sri Hariganesh Publications (Author: C. Ganesan)**

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Unit – I (Solution of Equations and Eigenvalue Problems)

• Fixed point iteration

1) Find a real root of the equation $x^3 + x^2 - 1 = 0$ by iteration method. (M/J 2012)

   Textbook Page No.: 1.1

2) Find the smallest positive root of $x^3 - 2x - 5 = 0$ by the fixed point iteration method, correct to three decimal places. (N/D 2017)

   Textbook Page No.: 1.3

3) Solve $e^x - 3x = 0$ by the method of fixed point iteration. (M/J 2012)

   Textbook Page No.: 1.4

4) Find a real root of the equation $x^3 + x^2 - 1 = 0$ by iteration method. (M/J 2012)

5) Find a positive root of the equation $\cos x - 3x + 1 = 0$ by using iteration method.

   Textbook Page No.: 1.6 (M/J 2013)

• Newton's method (or) Newton Raphson method

1) Solve for a positive root of the equation $x^4 - x - 10 = 0$ using Newton – Raphson method. (A/M 2010)

   Textbook Page No.: 1.8

2) Find by Newton-Raphson method a positive root of the equation $3x - \cos x - 1 = 0$.

   Textbook Page No.: 1.9 (N/D 2014)

3) Find an iterative formula to find the reciprocal of a given number N and hence find the value of $\frac{1}{19}$. (N/D 2011)

Textbook Page No.: 1.10

4) Find the Newton's iterative formula to calculate the reciprocal N and hence find the value of $\frac{1}{23}$. (N/D 2012),(A/M 2015)

Textbook Page No.: 1.12

5) Find an iterative formula to find $\sqrt{N}$, where N is a positive number and hence find $\sqrt{5}$. (N/D 2012)

Textbook Page No.: 1.12

- **Solution of linear system by Gaussian elimination method**

1) Apply Gauss elimination method to find the solution of the following system: $2x+3y-z=5,\ 4x+4y-3z=3,\ 2x-3y+2z=2$. (N/D 2012)

Textbook Page No.: 1.15

- **Solution of linear system by Gaussian-Jordon method**

1) Apply Gauss-Jordan method to find the solution of the following system:

$10x+y+z=12;\ 2x+10y+z=13;\ x+y+5z=7$. (N/D 2011)

Textbook Page No.: 1.18

2) Using Gauss-Jordan method to solve

$2x-y+3z=8;\ -x+2y+z=4;\ 3x+y-4z=0$. (N/D 2014)

Textbook Page No.: 1.20

3) Solve the system of equation by Gauss-Jordan method: $5x_1-x_2=9$, $-x_1+5x_2-x_3=4$, $-x_2+5x_3=-6$. (M/J 2014)

Textbook Page No.: 1.21

• Solution of linear system by Gaussian-Seidel method

1) Using Gauss-Seidel method, solve the following system of linear equations
$4x+2y+z=14$, $x+5y-z=10$, $x+y+8z=20$. (M/J 2014)

Textbook Page No.: 1.23

2) Apply Gauss-Seidal method to solve the equations
$20x+y-2z=17;\ 3x+20y-z=-18;\ 2x-3y+20z=25$.

Textbook Page No.: 1.25 (M/J 2012),(N/D 2014),(A/M 2018)

3) Solve, by Gauss-Seidel method, the following
system: $28x+4y-z=32;\ x+3y+10z=24;\ 2x+17y+4z=35$.

Textbook Page No.: 1.27 (N/D 2011),(N/D 2017),(N/D 2018)

4) Use Gauss – Seidel iterative method to obtain the solution of the equations:
$9x-y+2z=9;\ x+10y-2z=15;\ 2x-2y-13z=-17$. (A/M 2010)

Textbook Page No.: 1.29

5) Solve the following system of equations using Gauss-Seidel method:
$10x+2y+z=9;\ x+10y-z=-22;\ -2x+3y+10z=22$. (N/D 2012),(A/M 2015)

Textbook Page No.: 1.29

6) Solve, by Gauss-Seidel method, the following
system: $8x-3y+2z=20;\ 4x+11y-z=33;\ 6x+3y+12z=35$. (N/D 2012)

7) Solve, by Gauss-Seidel method, the
equations $27x+6y-z=85;\ 6x+15y+2z=72;\ x+y+54z=110$. (M/J 2013)

• Inverse of a matrix by Gauss Jordon method

1) Using Gauss-Jordan method, find the inverse of $\begin{bmatrix} 2 & 2 & 6 \\ 2 & 6 & -6 \\ 4 & -8 & 8 \end{bmatrix}$. (M/J 2014)

Textbook Page No.: 1.33

2) Using Gauss Jordan method, find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.(M/J 2012)

Textbook Page No.: 1.35

3) Find the inverse of the matrix by Gauss – Jordan method: $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$. (A/M 2010)

Textbook Page No.: 1.37

4) Using Gauss Jordan method, find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{bmatrix}$.

Textbook Page No.: 1.37 (N/D 2012),(A/M 2015)

5) Find, by Gauss-Jordan method, the inverse of the matrix $A = \begin{bmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix}$.

Textbook Page No.: 1.32 (M/J 2013),(N/D 2017)

- ## Eigen value of matrix by power method

1) Find the numerically largest eigenvalue of $A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ and its corresponding eigenvector by power method, taking the initial eigenvector as $(1 \quad 0 \quad 0)^T$.

(M/J 2014),(N/D 2014)

Textbook Page No.: 1.38

2) Determine the largest eigenvalue and the corresponding eigenvector of the matrix

$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. (M/J 2012)

Textbook Page No.: 1.44

3) Find the largest eigenvalue of $\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$, by using Power method.

(A/M 2010),(N/D 2011),(M/J 2012),(N/D 2012),(N/D 2017)

Textbook Page No.: 1.40

Unit – II (Interpolation and Approximation)

• Lagrange Polynomials and Divided differences

1) Using Lagrange's interpolation, calculate the profit in the year 2000 from the following data: (M/J 2012)

Year:	1997	1999	2001	2002
Profit Lakhs Rs.	43	65	159	248

Textbook Page No.: 2.1

2) Using Lagrange's interpolation formula find $y(9.5)$ for the given data (N/D 2018)

x:	7	8	9	10
y:	3	1	1	9

Textbook Page No.: 2.3

3) Using Lagrange's interpolation formula find the value of y when $x=10$, if the values of x and y are given below: (M/J 2012)

x:	5	6	9	11
y:	12	13	14	16

Textbook Page No.: 2.4

4) Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(3)$ for the following values of x and y: (N/D 2014)

x:	0	1	2	5
y:	2	3	12	147

(Or)

Find the interpolation polynomial $f(x)$ by Lagrange's formula and hence find $f(3)$ for $(0,2)$, $(1,3)$, $(2,12)$ and $(5,147)$. (N/D 2017)

Textbook Page No.: 2.5

5) Apply Lagrange's formula, to find $y(27)$ to the data given below. (M/J 2013),(A/M 2015)

x:	14	17	31	35
y:	68.8	64	44	39.1

6) Use Lagrange's formula to find a polynomial which takes the values $f(0)=-12,\ f(1)=0,\ f(3)=6$ and $f(4)=12$. Hence find $f(2)$. (A/M 2010)

Textbook Page No.: 2.6

7) Using Lagrange's interpolation formula, find $y(2)$ from the following data:
$y(0)=0;\ y(1)=1;\ y(3)=81;\ y(4)=256;\ y(5)=625$. (M/J 2014)

Textbook Page No.: 2.7

8) Use Lagrange's method to find $\log_{10} 656$, given that $\log_{10} 654 = 2.8156,$
$\log_{10} 658 = 2.8182,\ \log_{10} 659 = 2.8189$ and $\log_{10} 661 = 2.8202$. (N/D 2012)

Textbook Page No.: 2.9

9) Using Newton's divided difference formula, find the equation $y = f(x)$ of least degree and passing through the points $(-1,-21),(1,15),(2,12),(3,3)$. Find also y at $x = 0$.

Textbook Page No.: 2.10 (N/D 2018)

10) Determine $f(x)$ as a polynomial in x for the following data, using Newton's divided difference formulae. Also find $f(2)$. (N/D 2011)

x :	–4	–1	0	2	5
$f(x)$:	1245	33	5	9	1335

Textbook Page No.: 2.11

11) Find $f(3)$ by Newton's divided difference formula for the following data: (M/J 2014)

x :	–4	–1	0	2	5
$f(x)$:	1245	33	5	9	1335

Textbook Page No.: 2.11

12) Find the function $f(x)$ from the following table using Newton's divided difference formula: (A/M 2010)

x :	0	1	2	4	5	7
$f(x)$:	0	0	–12	0	600	7308

Textbook Page No.: 2.13

13) By using Newton's divided difference formula find $f(8)$, given (N/D 2014)

x :	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

Textbook Page No.: 2.14

14) Using Newton's divided differences formula determine $f(3)$ from the data:

x :	0	1	2	4	5
$f(x)$:	1	14	15	5	6

(M/J 2012)

Textbook Page No.: 2.18

15) Use Newton's divided difference formula to find $f(x)$ form the following data.

(M/J 2013),(A/M 2015)

x :	1	2	7	8
y :	1	5	5	4

Textbook Page No.: 2.17

16) Using Newton's divided difference formula, find $f(x)$ from the following data and hence find $f(4)$. (N/D 2012)

x	0	1	2	5
$f(x)$	2	3	12	147

Textbook Page No.: 2.16

- **Interpolating with a cubic spline**

1) Fit the cubic splines for the following data: (M/J 2014)

x :	1	2	3	4	5
y :	1	0	1	0	1

Textbook Page No.: 2.22

2) The following values of x and y are given:

x :	1	2	3	4
y :	1	2	5	11

Find the cubic splines and evaluate $y(1.5)$ and $y'(3)$. (M/J 2012)

Textbook Page No.: 2.19

3) From the following table:

x :	1	2	3
y :	- 8	-1	18

Compute $y(1.5)$ and $y'(1)$ using cubic sphere. (M/J 2012),(M/J 2013),(A/M 2015)

Textbook Page No.: 2.25

4) Obtain the cubic spline for the following data to find $y(0.5)$. (N/D 2012),(N/D 2014)

x :	-1	0	1	2
y :	-1	1	3	35

Textbook Page No.: 2.26

5) If $f(0)=1$, $f(1)=2, f(2)=33$ and $f(3)=244$. find a cubic spline approximation, assuming M(0) = M(3) = 0. Also, find $f(2.5)$. (A/M 2010)

6) Find the cubic spline approximation for the function give below. (N/D 2017)

x:	0	1	2	3
$f(x)$:	1	2	33	244

Assume that $M(0)=0=M(3)$. Hence find the value of $f(2.5)$.

Textbook Page No.: 2.26

- **Newton's forward and backward difference formulae**

1) Find the cubic polynomial which takes the following values:
(M/J 2012),(M/J 2013),(N/D 2014),(A/M 2015)

x:	0	1	2	3
$f(x)$:	1	2	1	10

Textbook Page No.: 2.27

2) Find the interpolation polynomial $f(x)$ by using Newton's forward difference interpolation formula and hence find the value of $f(5)$ for (N/D 2012),(N/D 2017)

x	4	6	8	10
y	1	3	8	16

Textbook Page No.: 2.28

3) Find the value of y at $x=21$ and $x=28$ from the data given below (N/D2012)

x:	20	23	26	29
y:	0.3420	0.3907	0.4384	0.4848

Textbook Page No.: 2.30

4) The population of a town is as follows:

x Year:	1941	1951	1961	1971	1981	1991
y Population in thousands:	20	24	29	36	46	51

Estimate the population increase during the period 1946 to 1976. (N/D 2011)

Textbook Page No.: 2.32

5) Given the following table, find the number of students whose weight is between 60 and 70 lbs: (A/M 2010), (M/J 2012)

Weight (in lbs) x:	0 – 40	40 – 60	60 – 80	80 – 100	100 – 120
No. of students:	250	120	100	70	50

Textbook Page No.: 2.34

Unit – III (Numerical Differentiation and Integration)

• Differentiation using interpolation formulae

1) From the following table find the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point $x = 0.96$.

(N/D 2018)

x:	0.96	0.98	1.00	1.02	1.04
y:	0.7825	0.7739	0.7651	0.7563	0.7473

Textbook Page No.: 3.2

2) The following data gives the corresponding values for pressure (p) and specific volume (v) of a superheated steam. Find the rate of change of pressure with respect to volume when $v = 2$. (N/D 2017)

v:	2	4	6	8	10
p:	105	42.7	25.3	16.7	13.0

Textbook Page No.: 3.4

3) A slider in a machine moves along a fixed straight rod. Its distance x cm along the rod is given below for various values of the time $'t'$ seconds. Find the velocity of the slider when $t = 1.1$ second. (M/J 2012)

t:	1.0	1.1	1.2	1.3	1.4	1.5	1.6
x:	7.989	8.403	8.781	9.129	9.451	9.750	10.031

(Or)

For the given data, find the first two derivatives at $x = 1.1$. (M/J 2014)

x:	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y:	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Textbook Page No.: 3.5

4) Find $f'(x)$ at $x = 1.5$ and $x = 4.0$ from the following data using Newton's formulae for differentiation. (N/D2012)

x :	1.5	2.0	2.5	3.0	3.5	4.0
$y = f(x)$:	3.375	7.0	13.625	24.0	38.875	59.0

(Or)

Find the first three derivatives of $f(x)$ at $x = 1.5$ by Newton's forward interpolation formula to the data given below. (M/J 2013),(A/M 2015)

x :	1.5	2.0	2.5	3.0	3.5	4.0
$y = f(x)$:	3.375	7.0	13.625	24.0	38.875	59.0

Textbook Page No.: 3.8

5) Given the following data, find $y'(6)$ and the maximum value of y (if it exists)

(A/M 2010)

x :	0	2	3	4	7	9
y :	4	26	58	112	466	922

6) Find the first two derivatives of $x^{1/3}$ at $x = 50$ and $x = 56$, for the given table: (N/D 2011)

x :	50	51	52	53	54	55	56
$y = x^{1/3}$	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

Textbook Page No.: 3.10

7) Find the first and second derivative of the function tabulated below at $x = 0.6$. (N/D 2012)

x :	0.4	0.5	0.6	0.7	0.8
y :	1.5836	1.7974	2.0442	2.3275	2.6511

8) Find the first and second derivatives of y with respect to x at $x = 10$ from the following data: (N/D 2017)

x :	3	5	7	9	11
y :	31	43	57	41	27

Textbook Page No.: 3.14

9) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 51$, from the following data: (M/J 2012)

x :	50	60	70	80	90
y :	19.96	36.65	58.81	77.21	94.61

Textbook Page No.: 3.12

- **Numerical integration using Trapezoidal, Simpson's 1/3, 3/8 rules & Romberg's method**

1) Evaluate $\int_0^1 \frac{dx}{1+x}$ and correct to 3 decimal places using Romberg's method and hence find the value of $\log_e 2$. (N/D 2014),(A/M 2018)

Textbook Page No.: 3.27

2) Use Romberg's method to compute $\int_0^1 \frac{dx}{1+x^2}$ correct to 4 decimal places. Also evaluate the same integral using tree-point Gaussian quadrature formula. Comment on the obtained values by comparing with the exact value of the integral which is equal to $\pi / 4$. (M/J 2012)

(Or)

Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Romberg's method correct to 4 decimal places. Hence deduce an approximate value for π. (M/J 2012)

(Or)

Using Romberg's integration to evaluate $\int_0^1 \frac{dx}{1+x^2}$. (A/M 2010)

Textbook Page No.: 3.29

3) Evaluate $\int_0^{\frac{1}{2}} \frac{x}{\sin x} dx$ correct to three decimal places using Romberg's method. (M/J 2014)

Textbook Page No.: 3.31

4) Using Trapezoidal rule, evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ by taking eight equal intervals. (M/J 2013)

Textbook Page No.: 3.16

5) Evaluate $\int_{0}^{1} \frac{x^2}{1+x^3}\, dx$ using Simpson's 1/3 rule by taking $h = 0.25$. (N/D 2018)

Textbook Page No.: 3.23

6) Compute $\int_{0}^{\pi/2} \sin x\, dx$ using Simpson's 3/8 rule. (N/D 2012)

Textbook Page No.: 3.19

7) Evaluate $I = \int_{0}^{2\pi} \sin x\, dx$ by dividing the range into ten equal parts, using

(i) Trapezoidal rule (N/D2012)

(ii) Simpson's one-third rule, Verify your answer with actual integration.

Textbook Page No.: 3.17

8) Evaluate $I = \int_{0}^{6} \frac{1}{1+x} dx$ by using (i) Direct integration (ii) Trapezoidal rule (iii) Simpson's one-third rule (iv) Simpson's three-eighth rule. (N/D 2011)

Textbook Page No.: 3.21

9) Taking $h = 0.05$, evaluate $\int_{1}^{1.3} \sqrt{x}\, dx$ using Trapezoidal rule and Simpson's three-eighth rule. (M/J 2014)

Textbook Page No.: 3.22

10) Using Simpson's one-third rule, evaluate $\int_{0}^{0.6} e^{-x^2} dx$ correct to three decimal places by step-size is 0.1. (N/D 2017)

Textbook Page No.: 3.24

11) The velocity υ of a particle at a distance S from a point on its path is given by the table below:

S (*meter*)	0	10	20	30	40	50	60
υ (*m / sec*)	47	58	64	65	61	52	38

Estimate the time taken to travel 60 meters by Simpson's 1/3rd rule and Simpson's 3/8th rule. (A/M 2010),(N/D 2014)

Textbook Page No.: 3.25

12) The velocities of a car running on a straight road at intervals of 2 minutes are given below: (A/M 2015)

Time (min):	0	2	4	6	8	10	12
Velocity (km/hr):	0	22	30	27	18	7	0

Using Simpson's 1/3 rd rule, find the distance covered by the car.

- **Two and Three point Gaussian Quadrature formulae**

1) Use Gaussian three-point formula and evaluate $\int_1^2 \frac{dx}{x}$. (N/D 2018)

Textbook Page No.: 3.37

2) Use Gaussian three-point formula and evaluate $\int_1^5 \frac{dx}{x}$. (M/J 2012)

Textbook Page No.: 3.38

3) Evaluate $\int_1^2 \frac{dx}{1+x^3}$ using 3 point Gaussian formula. (N/D 2014)

Textbook Page No.: 3.38

4) Evaluate $\int_0^2 \frac{x^2+2x+1}{1+(x+1)^2}dx$ by Gaussian three point formula. (M/J 2013)

Textbook Page No.: 3.40

5) Apply three points Gaussian quadrature formula to evaluate $\int_0^1 \frac{\sin x}{x}dx$. (A/M 2015)

Textbook Page No.: 3.42

• Double integrals by Trapezoidal and Simpsons's rules

1) Using Trapezoidal rule, evaluate $\int_1^2\int_1^2 \frac{dxdy}{x^2+y^2}$ numerically with $h=0.2$ along x -direction and $k=0.25$ along y -direction. (M/J 2012)

Textbook Page No.: 3.46

2) Evaluate $\int_0^1\int_0^1 \frac{1}{1+x+y}dxdy$ by trapezoidal rule. (N/D 2014)

Textbook Page No.: 3.48

3) Evaluate $\int_1^2\int_1^2 \frac{dxdy}{x+y}$ using Trapezoidal formula by taking $h=k=0.5$. (N/D 2018)

Textbook Page No.: 3.45

4) Evaluate $\int_1^{1.2}\int_1^{1.4} \frac{dxdy}{x+y}$ by trapezoidal formula by taking $h=k=0.1$. (A/M 2010)

Textbook Page No.: 3.50

5) Evaluate $\int_2^{2.4}\int_4^{4.4} xy\,dxdy$ by Trapezoidal rule taking $h=k=0.1$. (A/M 2015)

Textbook Page No.: 3.51

6) Evaluate $\int_0^2\int_0^1 4xy\ dxdy$ using Simpson's rule by taking $h=\frac{1}{4}$ and $k=\frac{1}{2}$.(N/D 2012)

7) Evaluate $\int_0^{1/2}\int_0^{1/2} \frac{\sin(xy)}{1+xy}dxdy$ using Simpson's rule with $h=k=\frac{1}{4}$.

Textbook Page No.: 3.55 (M/J 2012),(M/J 2014)

8) Evaluate $\int_2^{2.6}\int_4^{4.4} \frac{1}{xy}dxdy$ using Simpon's one-third rule with $h=0.2$ and $k=0.3$.

Textbook Page No.: 3.53 (A/M 2018)

9) Evaluate $\int_{1}^{1.2}\int_{2}^{2.4}\frac{1}{xy}dxdy$ using Simpon's one-third rule. (M/J 2013)

Textbook Page No.: 3.55

10) Evaluate $\int_{0}^{2}\int_{0}^{2} f(x,y)\,dxdy$ by Trapezoidal rule for the following data, correct to three decimal places: (N/D 2017)

y / x	0	0.5	1	1.5	2
0	2	3	4	5	5
1	3	4	6	9	11
2	4	6	8	11	14

Textbook Page No.: 3.44

Unit – IV (Initial Value Problems for Ordinary Differential Equations)

- **Taylor series method**

1) Use Tailor series method to find $y(0.1)$ and $y(0.2)$ given that $\frac{dy}{dx}=3e^{x}+2y,\ y(0)=0$, correct to 4 decimal accuracy. (A/M 2010)

Textbook Page No.: 4.2

2) Using Taylor series method solve $\frac{dy}{dx}=x^2-y,\ y(0)=1$ at $x=0.1,\ 0.2,\ 0.3$. Also compare the values with exact solution. (M/J 2012),(A/M 2015),(N/D 2017)

Textbook Page No.: 4.4

3) Using Taylor series method to find $y(0.1)$ if $y'=x^2+y^2,\ y(0)=1$.

Textbook Page No.: 4.1 (N/D 2011)(M/J 20113)

4) Using Taylor's series method, find y at $x=1.1$ by solving the equation $\frac{dy}{dx}=x^2+y^2;\ y(1)=2$. Carryout the computations upto fourth order derivative.

Textbook Page No.: 4.2 (M/J 2014)

5) Using Taylor's series method, find y at $x = 0$ if $\frac{dy}{dx} = x^2 y - 1$, $y(0) = 1$.

Textbook Page No.: 4.7 (N/D 2014),(N/D 2016)

• Euler method for first order equation

1) Solve $y' = \frac{y-x}{y+x}$, $y(0) = 1$ at $x = 0.1$ by taking $h = 0.02$ by using Euler's method.

Textbook Page No.: 4.9 (M/J 2013)

2) Find the value of y at $x = 0.1$ given that $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$ by modified Euler's method. (A/M 2018)

Textbook Page No.: 4.11

3) Apply modified Euler's method to find $y(0.2)$ and $y(0.4)$ given $y' = x^2 + y^2$, $y(0) = 1$ by taking $h = 0.2$. (N/D 2014),(A/M 2015)

Textbook Page No.: 4.12

4) Using Modified Euler's method, find $y(4.1)$ and $y(4.2)$ if

$5x\frac{dy}{dx} + y^2 - 2 = 0$; $y(4) = 1$. (N/D 2012)

Textbook Page No.: 4.15

• Fourth order Runge – Kutta method for 1st order equation

1) Find the value of $y(1.1)$ using Runge-Kutta method of fourth order given that

$\frac{dy}{dx} = y^2 + xy$, $y(1) = 1$. (N/D 2018)

Textbook Page No.: 4.19

2) Use Runge – Kutta method of fourth order to find $y(0.2)$, given $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$,

$y(0) = 1$, taking $h = 0.2$. (A/M 2010)

Textbook Page No.: 4.21

3) Given $y'' + xy' + y = 0,\ y(0) = 1,\ y'(0) = 0$ find the value of $y(0.1)$ by Runge-Kutta's method of fourth order. (A/M 2015)

Textbook Page No.: 4.24

- ## Milne's and Adam's predictor and corrector methods

1) Given $\frac{dy}{dx} = xy + y^2,\ y(0) = 1,\ y(0.1) = 1.1169$ and $y(0.2) = 1.2773$, find (i) $y(0.3)$ by R-K method of fourth order and (ii) $y(0.4)$ by Milne's method.

(N/D 2017),(A/M 2018)

Textbook Page No.: 4.31

2) Using Runge Kutta method of fourth order, find the value of y at $x = 0.2,\ 0.4,\ 0.6$ given $\frac{dy}{dx} = x^3 + y,\ y(0) = 2$. Also find the value of y at $x = 0.8$ using Milne's predictor and corrector method. (M/J 2014)

Textbook Page No.: 4.33

3) Given that $\frac{dy}{dx} = \frac{1}{2}\left(1 + x^2\right) y^2;\ y(0) = 1;\ y(0.1) = 1.06;\ y(0.2) = 1.12$ and $y(0.3) = 1.21$, evaluate $y(0.4)$ and $y(0.5)$ by Milne's predictor corrector method.

(N/D 2011)

Textbook Page No.: 4.28

4) Given that $\frac{dy}{dx} = 1 + y^2;\ y(0.6) = 0.6841,\ y(0.4) = 0.4228,\ y(0.2) = 0.2027,$ $y(0) = 0$, find $y(-0.2)$ using Milne's method. (N/D 2012)

Textbook Page No.: 4.38

5) Use Milne's predictor – corrector formula to find $y(0.4)$, given $\frac{dy}{dx} = \frac{\left(1 + x^2\right) y^2}{2}$, $y(0) = 1,\ y(0.1) = 1.06,\ y(0.2) = 1.12$ and $y(0.3) = 1.21$. (A/M 2010)

6) Given $y' = \frac{1}{x + y},\ y(0) = 2,\ y(0.2) = 2.0933,\ y(0.4) = 2.1755,\ y(0.6) = 2.2493$ find $y(0.8)$ using Milne's method. (M/J 2012)

Textbook Page No.: 4.38

7) Given $5xy' + y^2 = 2,\ y(4) = 1, y(4.1) = 1.0049,\ y(4.2) = 1.0097,\ y(4.3) = 1.0143$.
Compute $y(4.4)$ using Milne's method. (N/D 2014),(A/M 2015)
Textbook Page No.: 4.30

8) Using Adam's method, find $y(0.4)$ given $\frac{dy}{dx} = \frac{xy}{2}$, $y(0) = 1$, $y(0.1) = 1.01$, $y(0.2) = 1.002$ and $y(0.3) = 1.023$. (N/D 2017)

Textbook Page No.: 4.39

9) Given $\frac{dy}{dx} = xy + y^2,\ y(0) = 1,\ y(0.1) = 1.1169,\ y(0.2) = 1.2774,\ y(0.3) = 1.5041$.
Use Adam's method to estimate $y(0.4)$. (A/M 2010)

10) Using Adams method find $y(1.4)$ given $y' = x^2(1+y),\ \ y(1) = 1,\ \ y(1.1) = 1.233,$ $y(1.2) = 1.548$ and $y(1.3) = 1.979$. (M/J 2012)

Textbook Page No.: 4.41

11) Given that
$y' = y - x^2;\ y(0) = 1;\ y(0.2) = 1.1218;\ y(0.4) = 1.4682$ and $y(0.6) = 1.7379$,
evaluate $y(0.8)$ by Adam's predictor-corrector method. (N/D 2012)

Textbook Page No.: 4.47

12) Using Adam's method to find $y(2)$ if $y' = (x+y)/2,\ \ y(0) = 2,\ \ y(0.5) = 2.636,$ $y(1) = 3.595, y(1.5) = 4.968$. (M/J 2013)

Textbook Page No.: 4.47

13) Using Adam's Bashforth method, find $y(4.4)$ given that $5xy' + y^2 = 2,$ $y(4) = 1, y(4.1) = 1.0049, y(4.2) = 1.0097$ and $y(4.3) = 1.0143$. (M/J 2014)

Textbook Page No.: 4.42

Unit – V (Boundary Value Problems in ODE and PDE)

- **Finite Difference Solution of Second Order ODE**

1) Solve the equation $y'' = x + y$ with the boundary conditions $y(0) = y(1) = 0$ using finite differences by dividing the interval into four equal parts.
Textbook Page No.: 5.2 (M/J 2012),(M/J 2014),(A/M 2015)

2) Solve the boundary value problem $xy'' + y = 0$ with boundary conditions $y(1) = 1$ and $y(2) = 2$, taking $h = \frac{1}{4}$ by finite difference method. (N/D 2017)

Textbook Page No.: 5.4

3) Solve $y'' - y = 0$ with the boundary conditions $y(0) = 0$ and $y(1) = 1$. (N/D 2012)

Textbook Page No.: 5.1

4) Solve, by finite difference method, the boundary value problem $y''(x) - y(x) = 0$, where $y(0) = 0$ and $y(1) = 1$, taking $h = 0.25$. (M/J 2012)

Textbook Page No.: 5.1

5) Using the finite difference method, compute $y(0.5)$, given $y'' - 64y + 10 = 0,\ x \in (0,1), y(0) = y(1) = 0$, subdividing the interval into (i) 4 equal parts (ii) 2 equal parts. (N/D 2011),(N/D 2012)

Textbook Page No.: 5.5

- **One Dimensional Heat equation by explicit method**

1) Solve $u_t = u_{xx}$ in $0 < x < 4, t > 0$, given that $u(0,t) = 0$, $u(4,t) = 0$, $u(x,0) = x(4-x)$. Compute u upto $t = 4$ with $\Delta x = \Delta t = 1$. (N/D 2017)

Textbook Page No.: 5.8

2) Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, subject to $u(0,t) = u(1,t) = 0$ and $u(x,0) = \sin \pi x, \quad 0 < x < 1$, using Bender-Schmidt method. (M/J 2012)

Textbook Page No.: 5.10

3) Using Bender-Schmidt's method Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, given $u(0,t) = 0,\ u(1,t) = 0\ u(x,0) = \sin \pi x, \quad 0 < x < 1$ and $h = 0.2$. Find the value of u upto $t = 0.1$. (M/J 2014),(A/M 2015)

Textbook Page No.: 5.11

4) Solve by Bender-Schmidt formula upto $t = 5$ for the equation $u_{xx} = u_t$, subject to $u(0,t) = 0,\ u(5,t) = 0,$ and $u(x,0) = x^2\left(25 - x^2\right)$, taking $h = 1$. (N/D 2012)

Textbook Page No.: 5.11

5) Solve $\frac{\partial u}{\partial t}=\frac{1}{2}\frac{\partial^2 u}{\partial x^2}$ with the condition $u(0,t)=0=u(4,t)$, $u(x,0)=x(4-x)$ taking $h=1$ employing Bender-Schmidt recurrence equation. Continue the solution through 10 time steps. (M/J 2012)

Textbook Page No.: 5.9

6) Solve $u_{xx}=32u_t,\ h=0.25$ for $t\geq 0,\ 0<x<1,\ u(0,t)=0,\ u(x,0)=0, u(1,t)=t$.

Textbook Page No.: 5.13 (M/J 2013)

- **One Dimensional Heat equation by implicit method**

1) Obtain the Crank – Nicholson finite difference method by taking $\lambda=\frac{kc^2}{h^2}=1$. Hence, find $u(x,t)$ in the rod for two times steps for the heat equation $\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}$, given $u(x,0)=\sin(\pi x),\ u(0,t)=0,\ u(1,t)=0$. Take h = 0.2. (A/M 2010)

Textbook Page No.: 5.16

2) Solve by Crank-Nicolson's method $\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}$ for $0<x<1,\ t>0$ given that $u(0,t)=0,\ u(1,\text{t})=0$ and $u(x,0)=100x(1-x)$. Compute u for one time step with $h=\frac{1}{4}$ and $k=\frac{1}{64}$. (N/D 2014)

Textbook Page No.: 5.14

- **One Dimensional Wave equation**

1) Solve the equation $\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2},\ 0<x<1,\ t>0$ satisfying the conditions $u(x,0)=0,\ \frac{\partial u}{\partial t}(x,0)=0,\ u(0,t)=0$ and $u(1,t)=\frac{1}{2}\sin\pi t$. Compute $u(x,t)$ for 4 time-steps by taking $h=\frac{1}{4}$. (N/D 2012)

Textbook Page No.: 5.20

2) Solve the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \; 0 < x < 1, \; t > 0$,

$u(0,t) = u(1,t) = 0, \text{t} > 0 \;\; u(x,0) = \begin{cases} 1, & 0 \le x \le 0.5 \\ -1, & 0.5 \le x \le 1 \end{cases}$ and $\frac{\partial u}{\partial t}(x,0) = 0,$ using

$h = k = 0.1$, find u for three time steps. (M/J 2014),(A/M 2015)

Textbook Page No.: 5.21

3) Find the pivotal values of the equation $\frac{1}{4}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with given conditions

$u(0,t) = 0, \;\; u(4,t) = 0$, $u(x,0) = x(4-x)$ and $\frac{\partial u}{\partial t}(x,0) = 0$ by taking $h = 1$, for 4 time steps. (M/J 2012)

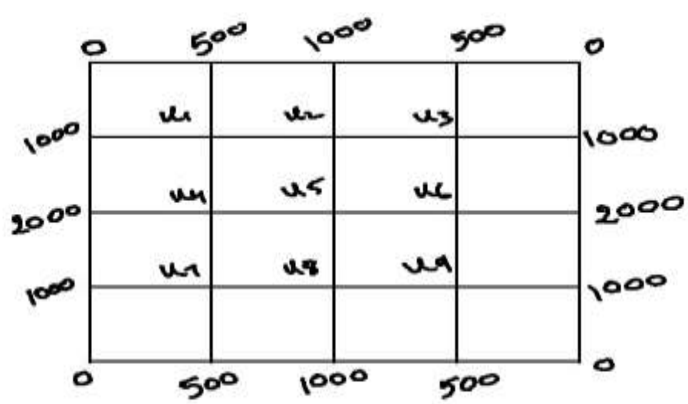
(Or)

Solve $4u_{tt} = u_{xx}, \; u(0,t) = 0, \; u(4,t) = 0$, $u(x,0) = x(4-x), \; u_t(x,0) = 0, \; h = 1$ upto $t = 4$. (M/J 2013)

Textbook Page No.: 5.23

• Two Dimensional Laplace and Poisson equations

1) Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown: (M/J 2012)



Textbook Page No.: 5.24

2) By iteration method, solve the elliptic equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ over a square region of side 4, satisfying the boundary conditions.

(i) $u(0,y)=0,\quad 0\le y\le 4$ (ii) $u(4,y)=12+y,\quad 0\le y\le 4$

(ii) $u(x,0)=3x,\quad 0\le x\le 4$ (iv) $u(x,4)=x^2,\quad 0\le x\le 4$

By dividing the square into 16 square meshes of side 1 and always correcting the computed values to two places of decimals, obtain the values of u at 9 interior pivotal points. (N/D 2014),(A/M 2018),(N/D 2018)

Textbook Page No.: 5.30

3) Solve the Poisson equation $\nabla^2 u=-10\left(x^2+y^2+10\right)$ over the square mesh with sides $x=0,\ y=0,\ x=3$ and $y=3$ with $u=0$ on the boundary and mesh length 1 unit. (N/D 2012),(M/J 2014),(A/M 2015),(N/D 2017)

Textbook Page No.: 5.34

4) Solve $\nabla^2 u=8x^2y^2$ for square mesh given $u=0$ on the four boundaries dividing the square into 16 sub-squares of length 1 unit. (N/D 2011)

(Or)

Solve $\nabla^2 u=8x^2y^2$ over the square $x=-2,\ x=2,\ y=-2,\ y=2$ with $u=0$ on the boundary and mesh length = 1. (M/J 2013)

Textbook Page No.: 5.36

Book for Reference:

"Numerical Methods"

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----All the Best----