**NAME OF THE SUBJECT** : **Engineering Mathematics – I**

**SUBJECT CODE** : **MA8151**

**MATERIAL NAME** : **Anna University Question Bank**

**REGULATION** : **R 2017**

**WEBSITE** : **www.hariganesh.com**

**UPDATED ON** : **8th March 2020**

**TEXT BOOK FOR REFERENCE** : **Sri Hariganesh Publications (Author: C. Ganesan)**

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# Unit – I (Differential Calculus)

## • Problems on Limits and Continuous function

1. Find the values of $a$ and $b$, if the function $f$ defined by $f(x)=\begin{cases}3ax+b & if\ \ x<1\\ 11 & if\ \ x=1\\ 5ax-2b & if\ \ x>1\end{cases}$ is continuous at $x=1$. Textbook Page No.: 1.21 Video Explanations

2. For what value of the constant "$b$" is the function "$f$" continuous on $(-\infty,\infty)$,
$f(x)=\begin{cases}bx^2+2x; & x<2\\ x^3-bx; & x\le 2\end{cases}$. (Jan 2018),(A/M 2019)
Textbook Page No.: 1.24 Video Explanations

3. Find the values of $a$ and $b$ such that the function defined by
$f(x)=\begin{cases}5 & if\ \ x\le 2\\ ax+b & if\ \ 2<x<10\\ 21 & if\ \ x\ge 10\end{cases}$ is a continuous function.
Textbook Page No.: 1.24

4. Find the values of $a$ and $b$, if the function $f(x)=\begin{cases}x^2+3x+a & \text{if } x\le 1\\ bx+2 & \text{if } x>1\end{cases}$ is differentiable at $x=1$. Textbook Page No.: 1.29

5. Find the values of $a$ and $b$ that make $f$ continuous on $(-\infty, \infty)$

$$f(x) = \begin{cases} \dfrac{x^3-8}{x-2} & \text{if } x<2 \\ ax^2-bx+3 & \text{if } 2\le x<3 \\ 2x-a+b & \text{if } x\ge 3 \end{cases}.$$

(N/D 2018)

Textbook Page No.: 1.25

6. If the function $f(x) = \begin{cases} x^2 & \text{if } x\le 2 \\ ax+b & \text{if } x>2 \end{cases}$ is differentiable at $x=2$, then find the values of $a$ and $b$.

Textbook Page No.: 1.31 Video Explanations

7. Show that the function $f(x) = \begin{cases} x & \text{if } x<1 \\ 2-x & \text{if } 1\le x\le 2 \\ -x^2+3x-2 & \text{if } x>2 \end{cases}$ is not differentiable at $x=1$ but it is differentiable at $x=2$.

Textbook Page No.: 1.32

8. Show that the function $f(x) = \begin{cases} 3x-2 & \text{if } 0\le x\le 1 \\ 2x^2-x & \text{if } 1<x\le 2 \\ 5x-4 & \text{if } x>2 \end{cases}$ is continuous at $x=1$ and $x=2$ but not differentiable at $x=2$.

Textbook Page No.: 1.33 Video Explanations

- **Derivatives of Elementary Functions**

1. If $f(x) = \dfrac{1-x}{2+x}$ then, find the equation for $f'(x)$ using the concept of derivatives.

Video Explanation (N/D 2019)

2. Find $\dfrac{dy}{dx}$ for the following functions

   i) $y = 2x^4 - 3x^3 + 12x^2 + 5$

   ii) $y = e^{-x} + \log x$

   iii) $y = \dfrac{x^3 - 2x^2 + 5}{x^2}$

   iv) $y = e^x + 3\tan x + \log x^4$

   v) $y = \sin 3 + \log_{10} x + 2\sec x$

3. Find $\frac{dy}{dx}$ for the following functions

i) $y=\left(4x^2-3\right)\left(2x+1\right)$

ii) $y=\left(x^2+7x+2\right)\left(e^x-\log x\right)$

iii) $y=e^x\sin x$

iv) $y=\left(3\sec x-4\cos ecx\right)\left(2\sin x+5\cos x\right)$

v) $y=x^2e^x\sin x$

vi) $y=\sqrt{x}\,e^x\log x$

4. Find $\frac{dy}{dx}$ for the following functions

i) $y=\frac{2x+3}{3x-5}$

ii) $y=\frac{\cos x+\log x}{x^2+e^x}$

iii) $y=\frac{\sin x+\cos x}{\sin x-\cos x}$

iv) $y=\frac{\tan x+1}{\tan x-1}$

v) $y=\frac{x^2+e^x\sin x}{\cos x+\log x}$

vi) $y=\frac{\sin x+x\cos x}{x\sin x-\cos x}$

5. Find $\frac{dy}{dx}$ for the following functions

i) $y=\sin\left(x^2+2x+3\right)$

ii) $y=e^{\sin x}$

iii) $y=\tan\left(\log x\right)$

iv) $y=\sqrt{1+\cot x}$

v) $y=\log\sqrt{x}$

vi) $y=e^{\sin x^2}$

vii) $y=e^{\sin(\log x)}$

6. Find $\frac{dy}{dx}$ for the following functions

i) $x=at^2,\ y=2at$

ii) $x=a\cos\theta,\ y=b\sin\theta$

iii) $x=ct,\ y=\frac{c}{t}$

iv) $x=a\left(\theta-\sin\theta\right),\ y=a\left(1-\cos\theta\right)$

7. Find $\frac{dy}{dx}$ for the following functions

i) $x^3+8xy+y^3=64$

ii) $x^3+y^3=3axy$

iii) $e^x+e^y=e^{x+y}$

iv) $(1+y^2)\sec x - y\cot x + 1 = x^2$

v) $ax^2+by^2+2gx+2fy+2hxy+c=0$

vi) Find $y'$ for $\cos(xy)=1+\sin y$. (N/D 2018)

Textbook Page No.: 1.69

8. Find $\frac{dy}{dx}$ if $y=\cos^{-1}\left(\frac{b+a\cos x}{a+b\cos x}\right)$. (N/D 2018)

Textbook Page No.: 1.75

9. Find the derivative of $f(x)=\tanh^{-1}\left[\tan\frac{x}{2}\right]$. (N/D 2019)

Video Explanation

10. Find $y''$ if $x^4+y^4=16$. (Jan 2018)

Textbook Page No.: 1.70

- **Tangent and Normal to the curve**

1. Find the equation of the tangent and normal to the curve $y=x^3$ at the point $(2, 8)$.

Textbook Page No.: 1.101

2. Find the equation of the tangent and normal to the curve $y=x^4+2e^x$ at the point $(0, 2)$.

Textbook Page No.: 1.103

3. Find the equation of the tangent and normal to the curve $y=x\sqrt{x}$ at the point $(1, 1)$.

Textbook Page No.: 1.104

4. Find the tangent line to the equation $x^3+y^3=6xy$ at the point $(3,3)$ and at what point the tangent line horizontal in the first quadrant. (Jan 2018)

Textbook Page No.: 1.107

## • Derivative using Logarithmic

1. Find $\frac{dy}{dx}$, when $y=(\tan x)^{\log x}$.

2. Find $\frac{dy}{dx}$, when $y=(\tan x)^{\cot x}+(\cot x)^{\tan x}$.

3. Find $\frac{dy}{dx}$, when $y=\frac{(1-x)\sqrt{x^2+2}}{(x+3)\sqrt{x-1}}$.

4. Find $\frac{dy}{dx}$, when $y=(x^2+x+1)^{\sqrt{x-1}}$.

5. Find $\frac{dy}{dx}$ if $y=x^2e^{2x}(x^2+1)^4$. (A/M 2019)

   Textbook Page No.: 1.78

6. Find $\frac{dy}{dx}$, when $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+\ldots\infty}}}$.

## • Maxima and Minima of functions of one variable

1. Maxima and Minima – Definition and Working Rule. Video Explanations

2. Find where the function $f(x)=3x^4-4x^3-12x^2+5$ is increasing and decreasing and also find the local maximum and minimum values.

   Textbook Page No.: 1.117 Video Explanations

3. Find the local maximum and local minimum values of the function $f$ given by $f(x)=3x^4+4x^3-12x^2+12$. Using both first and second derivative test.

   Textbook Page No.: 1.112

4. If $f(x)=2x^3+3x^2-36x$, find the intervals on which it is increasing or decreasing, the local maximum and local minimum value of $f(x)$. Also find the intervals of concavity and the inflection points. (A/M 2019),(N/D 2019)

   Textbook Page No.: 1.114

5. If $f(x)=2+2x^2-x^4$, find the intervals of increase or decrease, local maximum and minimum values, the intervals of concavity and the inflection points. (N/D 2018)

Textbook Page No.: 1.115

6. Find the maximum and minimum values of $f(x) = x^4 - 3x^3 + 3x^2 - x$.

7. Find the absolute maximum and minimum values of a function $f$ given by $f(x) = 2x^3 - 15x^2 + 36x + 1$ on the interval $[1, 5]$.

Textbook Page No.: 1.120

8. Find the local maximum and minimum values of $f(x) = \sqrt{x} - \sqrt[4]{x}$ using both the first and second derivative tests. (Jan 2018)

Textbook Page No.: 1.117

9. Find the local maximum and minimum values of the function $f(x) = x + 2\sin x, \quad 0 \le x \le 2\pi$.

Textbook Page No.: 1.120

# Unit – II (Functions of Several Variables)

- **Partial Differentiation**

1. If $u = \left(x^2 + y^2 + z^2\right)^{-1/2}$ then find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$. (Jan 2018)

Textbook Page No.: 2.4

2. If $u = \log\left(x^2 + y^2\right) + \tan^{-1}\left(y/x\right)$ prove that $u_{xx} + u_{yy} = 0$. (Jan 2009),(N/D 2010)

Textbook Page No.: 2.5 Video Explanation

3. For the given function $z = \tan^{-1}\left(\frac{x}{y}\right) - (xy)$, verify whether the statement $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$, is correct or not. Video Explanation (N/D 2019)

4. If $u = \log\left(\tan x + \tan y + \tan z\right)$, find $\sum \sin 2x \frac{\partial u}{\partial x}$. (M/J 2015)

Video Explanation

## • Euler's theorem and Total derivatives

1. If $u=\cos^{-1}\left[\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right]$, then prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\dfrac{-1}{2}\cot u$. (A/M 2017)

   Textbook Page No.: 2.8

2. If $u=f(r,s,t)$ and $r=\dfrac{x}{y}$, $s=\dfrac{y}{z}$, $t=\dfrac{z}{x}$, find the value of $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}+z\dfrac{\partial u}{\partial z}$.

   Textbook Page No.: 2.10 (N/D 2016)

3. If $w=f\left(y-z,z-x,x-y\right)$, then show that $\dfrac{\partial w}{\partial x}+\dfrac{\partial w}{\partial y}+\dfrac{\partial w}{\partial z}=0$.

   Textbook Page No.: 2.15 Video Explanation (Jan 2014),(Jan 2016),(M/J 2016)

4. If $u=f(2x-3y,\,3y-4z,\,4z-2x)$, then find $\dfrac{1}{2}\dfrac{\partial u}{\partial x}+\dfrac{1}{3}\dfrac{\partial u}{\partial y}+\dfrac{1}{4}\dfrac{\partial u}{\partial z}$. (A/M 2019)

   Textbook Page No.: 2.16

5. If $u=f\left(\dfrac{y-x}{xy},\dfrac{z-x}{xz}\right)$, find $x^2\dfrac{\partial u}{\partial x}+y^2\dfrac{\partial u}{\partial y}+z^2\dfrac{\partial u}{\partial z}$. (N/D 2018)

   Textbook Page No.: 2.17 Video Explanations

6. If $z=f(x,y)$, where $x=u^2-v^2, y=2uv$, prove that

   $\dfrac{\partial^2 z}{\partial u^2}+\dfrac{\partial^2 z}{\partial v^2}=4\left(u^2+v^2\right)\left(\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial y^2}\right)$. (Jan 2010),(Jan 2012)

   Textbook Page No.: 2.20

7. If $u=f(x,y)$ where $x=r\cos\theta,\ y=r\sin\theta$, prove that

   $\left(\dfrac{\partial u}{\partial x}\right)^2+\left(\dfrac{\partial u}{\partial y}\right)^2=\left(\dfrac{\partial u}{\partial r}\right)^2+\dfrac{1}{r^2}\left(\dfrac{\partial u}{\partial \theta}\right)^2$. (M/J 2010)

   Textbook Page No.: 2.18 Video Explanations

- **Taylor's Series**

1. Find the Taylor's series expansion of $x^2y^2+2x^2y+3xy^2$ in powers of $(x+2)$ and $(y-1)$ upto 3rd degree terms. (Jan 2010),(M/J 2010),(Jan 2012),(A/M 2019)

   Textbook Page No.: 2.39

2. Use Taylor's formula to expand the function defined by $f(x,y)=x^3+y^3+xy^2$ in powers of $(x-1)$ and $(y-2)$. (A/M 2011),(M/J 2015),(A/M 2017),(Jan 2018)

   Textbook Page No.: 2.41

3. Expand $x^2y+3y-2$ in powers of $(x-1)$ and $(y+2)$ upto 3rd degree terms.

   Video Explanation (M/J 2012)

4. Find the Taylor series expansion of $e^x \sin y$ at the point $(-1, \pi/4)$ up to 3rd degree terms. (Jan 2009),(M/J 2009)

   Textbook Page No.: 2.46 Video Explanations

5. Expand $e^x \sin y$ in powers of $x$ and $y$ as far as the terms of the 3rd degree using Taylor's expansion. (M/J 2013),(Jan 2016),(N/D 2016),(N/D 2019)

   Textbook Page No.: 2.43

6. Find the Taylor's series expansion of $e^x \cos y$ in the neighborhood of the point $\left(1,\frac{\pi}{4}\right)$ upto third degree terms. Video Explanation (N/D 2010)

7. Expand $e^x \cos y$ at $\left(0,\frac{\pi}{2}\right)$ upto the third term using Taylor's series. (M/J 2014)

8. Expand $e^x \log(1+y)$ in power of $x$ and $y$ upto terms of third degree using Taylor's theorem. (N/D 2011),(Jan 2014),(M/J 2016)

   Textbook Page No.: 2.48

9. Expand $\sin xy$ at $\left(1,\frac{\pi}{2}\right)$ upto second degree terms using Taylor's series. (N/D 2014)

   Textbook Page No.: 2.50

10. Find the Taylor's series expansion of $f(x,y)=\sqrt{1+x+y^2}$ in powers of $(x-1)$ and $y$ up to the second degree terms. (N/D 2018)
    Textbook Page No.: 2.54

## • Maxima and Minima of functions of two variables

1. Find the extreme values of the function $f(x,y)=x^3+y^3-3x-12y+20$.
   Textbook Page No.: 2.57 (Jan 2010),(A/M 2011),(Jan 2012),(N/D 2014)
   Video Explanations
2. Test for maxima and minima of the function $f(x,y)=x^3+y^3-12x-3y+20$.
   (M/J 2013)
3. Find the maximum or minimum values of $f(x,y)=3x^2-y^2+x^3$. (Jan 2018)
   Textbook Page No.: 2.58
4. Examine $f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$ for extreme values.
   Textbook Page No.: 2.60 (Jan 2016),(A/M 2019)
5. Find the maximum and minimum values of $x^2-xy+y^2-2x+y$. (M/J 2012)
   Textbook Page No.: 2.59
6. Find the maximum or minimum values of the function $f(x,y)=x^2+y^2+6x+12$.
   (N/D 2019)
7. Discuss the maxima and minima of the function $f(x,y)=x^4+y^4-2x^2+4xy-2y^2$.
   Textbook Page No.: 2.62 (N/D 2010),(N/D 2018)
8. Examine the extrema of $f(x,y)=x^2+xy+y^2+\frac{1}{x}+\frac{1}{y}$. (Jan 2014),(N/D 2016)
   Textbook Page No.: 2.64
9. Examine the function $f(x,y)=x^3y^2(12-x-y)$ for extreme values. (M/J 2009)
   Textbook Page No.: 2.66
10. Test for the maxima and minima of the function $f(x,y)=x^3y^2(6-x-y)$.(Jan 2013)
11. Discuss the maxima and minima of $f(x,y)=x^3y^2(1-x-y)$. (Jan 2014)
    Textbook Page No.: 2.68
12. Find the minimum values of $x^2yz^3$ subject to the condition $2x+y+3z=a$.
    Text Book Page No.: 4.87 (A/M 2017)
13. A thin closed rectangular box is to have one edge equal to twice the other and constant volume 72 m$^3$. Find the least surface area of the box. (N/D 2019)
    Video Explanation

14. A rectangular box open at the top, is to have a volume of 32 cc. Find the dimensions of the box, that requires the least material for its construction.
Textbook Page No.: 2.74 (M/J 2010),(N/D 2011),(M/J 2012),(M/J 2016),(A/M 2017)

15. A rectangular box open at the top, is to have a capacity of 108 cu. ms. Find the dimensions of the box requiring the least material for its construction.
Textbook Page No.: 2.80 (Jan 2014), (Jan 2018)

16. Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface area is 432 square meter. (M/J 2013)
Textbook Page No.: 2.77

17. Find the volume of the greatest rectangular parallelepiped inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. (M/J 2009),(M/J 2015)
Textbook Page No.: 2.82

18. Find the shortest and longest distances from the point $(1,2,-1)$ to the sphere $x^2+y^2+z^2=24$. (N/D 2016),(A/M 2019)
Textbook Page No.: 2.91

- **Jacobians**

1. Find the Jacobian $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ of the transformation $x=r\sin\theta\cos\phi,\ y=r\sin\theta\sin\phi$ and $z=r\cos\theta$. (Jan 2009),(A/M 2011),(Jan 2016),(M/J 2016)

   Textbook Page No.: 2.34 Video Explanations

2. If $x+y+z=u,\ y+z=uv,\ z=uvw$ prove that $\frac{\partial(x,y,z)}{\partial(u,v,w)}=u^2v$.

   Textbook Page No.: 2.35 (Jan 2010),(Jan 2012)

3. Find the Jacobian of $y_1,y_2,y_3$ with respect to $x_1,x_2,x_3$ if $y_1=\frac{x_2x_3}{x_1},y_2=\frac{x_3x_1}{x_2},\ y_3=\frac{x_1x_2}{x_3}$.

   Textbook Page No.: 2.36 (N/D 2010)

# Unit – III (Integral Calculus)

## • Definite and Indefinite Integrals

1. Evaluate $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{3/2}}{(\sin x)^{3/2}+(\cos x)^{3/2}}dx$. Video Explanation

2. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$. Textbook Page No.: 3.101

3. Evaluate $\int_0^{\frac{\pi}{2}} \log(\tan x)\,dx$. Textbook Page No.: 3.102

4. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$. Textbook Page No.: 3.106

5. Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan x}$. (A/M 2019) Textbook Page No.: 3.107

6. Evaluate $\int_0^{2} \frac{\sqrt{x}\,dx}{\sqrt{x}+\sqrt{2-x}}$. Textbook Page No.: 3.109

7. Evaluate $\int \frac{\tan x}{\sec x+\cos x}dx$. Video Explanation (Jan 2018)

## • Simple Problems on Integral Calculus

1. Evaluate $\int \frac{x^2-5x+1}{x}dx$.

2. Evaluate $\int (2x-5)(4+2x)\,dx$.

3. Evaluate $\int \frac{e^x+1}{e^x}dx$.

4. Evaluate $\int \sec x\,dx$.

5. Evaluate $\int \frac{1-\tan x}{1+\tan x}dx$.

6. Evaluate $\int \tan^{-1} x\,dx$.

7. Evaluate $\int \frac{\tan^{-1} x}{1+x^2}dx$.

8. Evaluate $\int \sqrt{a^2-x^2}\,dx$ by using substitution rule. (N/D 2019)

   Textbook Page No.: 3.23

## Integration by Parts

1. Evaluate $\int e^{ax}\cos bx\,dx$ using integration by parts. (Jan 2018)

   Textbook Page No.: 3.38

2. Evaluate $\int e^{x}\sin x\,dx$ by using integration by parts. (N/D 2019)

3. Using integration by parts, evaluate $\int \frac{(\ln x)^2}{x^2}dx$. (N/D 2018)

   Video Explanations

## Problems on Reduction Formula

1. Evaluate $\int \sin^n x\,dx$. Textbook Page No.: 3.45

2. Evaluate $\int \cos^n x\,dx$. Textbook Page No.: 3.47

3. Evaluate $\int_0^{\frac{\pi}{2}} \cos^5 x\,dx$. (Jan 2018)

4. Evaluate $\int \sin^m x\cos^n x\,dx$. Textbook Page No.: 3.49

5. Evaluate $\int_0^{\pi} \sin^2 x\,\cos^4 x\,dx$. (N/D 2019)

## Integration of Rational and Irrational Functions

1. Introduction and Simple Problems. Video Explanation

2. Evaluate $\int \frac{x+2}{x^2-4x+3}dx$. Video Explanation Textbook Page No.: 3.55

3. Evaluate $\int \frac{x+1}{(x-2)^2(x+3)}dx$. Textbook Page No.: 3.57

4. Evaluate $\int \frac{dx}{(x^2+4)(x+1)}$. Textbook Page No.: 3.58

5. Evaluate $\int_0^{\pi/2} \frac{\sin x\cos x}{\cos^2 x+3\cos x+2}dx$. (A/M 2019)

   Textbook Page No.: 3.60

6. Evaluate $\int \frac{dx}{x^2+3x-3}$. Textbook Page No.: 3.64

7. Evaluate $\int \frac{3x+5}{x^2+4x+7}dx$. Video Explanation

8. Evaluate $\int \frac{3-2x}{x^2+x+1}dx$. Textbook Page No.: 3.67

9. Evaluate $\int \sqrt{x^2-3x+10}\,dx$. Textbook Page No.: 3.74

10. Evaluate $\int \sqrt{1+2x-3x^2}\,dx$. Textbook Page No.: 3.74 Video Explanation
11. Evaluate $\int (3x-2)\sqrt{x^2+x+1}\,dx$. Textbook Page No.: 3.75 Video Explanation
12. Evaluate $\int \frac{dx}{\sqrt{2x^2+3x+4}}$. Textbook Page No.: 3.95 Video Explanation
13. Evaluate $\int \frac{dx}{\sqrt{x^2+3x+10}}$. Textbook Page No.: 3.79
14. Evaluate $\int \frac{dx}{\sqrt{1+x-x^2}}$. Textbook Page No.: 3.80
15. Evaluate $\int \frac{x+2}{\sqrt{x^2+2x-3}}\,dx$. Video Explanation
16. Evaluate $\int \frac{2x+5}{\sqrt{x^2-2x+10}}\,dx$. Textbook Page No.: 3.87 (A/M 2019)
17. Evaluate $\int \frac{x}{\sqrt{x^2+x+1}}\,dx$. Textbook Page No.: 3.93 (Jan 2018)
18. Evaluate $\int \frac{2x-3}{\sqrt{10-7x-x^2}}\,dx$. Textbook Page No.: 3.89
19. Evaluate $\int \frac{dx}{(2-x)\sqrt{1-2x+3x^2}}$. Video Explanation

# Unit – IV (Multiple Integrals)

- **Double Integration**

1. Basic Problems in Double Integrations. Video Explanations
2. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2}\,dxdy$. (N/D 2016)

   Textbook Page No.: 4.6

- **Change of Order of Integration**

1. Basic Problems. Video Explanations
2. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dxdy$ by changing the order of integration.

   Textbook Page No.: 4.21 (N/D 2010),(A/M 2011),(A/M 2019)

3. Change the order of integration for the given integral $\int_0^a \int_0^{2\sqrt{ax}} x^2 dydx$ and evaluate it.

Video Explanation (Jan 2018)

4. Change the order of integration in $\int_0^2 \int_0^{\sqrt{4-y^2}} xy\ dxdy$ and evaluate it. (N/D 2016)

Textbook Page No.: 4.33

5. Change the order of integration and hence evaluate it $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy\ dydx$. (A/M 2017)

Textbook Page No.: 4.22

6. Change the order of integration $\int_0^1 \int_{x^2}^{2-x} xy\ dxdy$ and hence evaluate.

Video Explanations (Jan 2010),(M/J 2012),(Jan 2014),(Jan 2016),(M/J 2016)

7. Change the order of integration in the interval $\int_0^a \int_{x^2/a}^{2a-x} xy\ dydx$ and hence evaluate it.

Textbook Page No.: 4.28 (M/J 2010),(Jan 2013),(M/J 2014)

8. Change the order of integration and hence find the value of $\int_0^1 \int_y^{2-y} xy\ dxdy$ .(N/D 2011)

Textbook Page No.: 4.35

9. Change the order of integration in $\int_0^a \int_y^a \frac{x}{x^2+y^2}\ dxdy$ and hence evaluate it. (M/J 2013)

Textbook Page No.: 4.20

10. By changing the order of integration, evaluate $\int_0^1 \int_y^1 \frac{x}{x^2+y^2}\ dxdy$ .(M/J 2015)

Video Explanation

## • Double Integrals in Polar Coordinates

1. Evaluate by changing to polar coordinates $\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dxdy$. (Jan 2018)

   Textbook Page No.: 4.52

2. Express $\int_0^a \int_y^a \frac{x^2\,dxdy}{\left(x^2+y^2\right)^{3/2}}$ in polar coordinates and then evaluate it.

   Textbook Page No.: 4.56 Video Explanations (M/J 2009),(N/D 2018)

3. Evaluate $\int_0^\infty \int_0^\infty e^{-\left(x^2+y^2\right)}\,dxdy$ by converting to polar coordinates. Hence deduce the value of $\int_0^\infty e^{-x^2}\,dx$. (Jan 2010),(N/D 2010),(Jan 2014),(Jan 2016),(M/J 2016),(N/D 2016)

   Textbook Page No.: 4.64 Video Explanations

4. Transform the integral $\int_0^2 \int_0^{\sqrt{2x-x^2}} \left(x^2+y^2\right)dydx$ into polar coordinates and hence evaluate it. (A/M 2011),(N/D 2014)

   Textbook Page No.: 4.58

5. By Transforming into polar coordinates, evaluate $\iint \frac{x^2y^2}{x^2+y^2}\,dxdy$ over annular region between the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$, $(b>a)$. (Jan 2013)

   Textbook Page No.: 4.66

6. Transform the double integral $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{dxdy}{\sqrt{a^2-x^2-y^2}}$ into polar co-ordinates and then evaluate it. (Jan 2012)

   Textbook Page No.: 4.61

7. Transform the integral into polar coordinates and hence evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\,dydx$.

   Textbook Page No.: 4.60 (Jan 2012)

## • Area enclosed by Plane Curves

1. Find the area of the circle using double integrations. Video Explanations
2. Using double integral, find the area bounded by $y = x$ and $y = x^2$. (Jan 2018)

   Textbook Page No.: 4.40
3. Find, by double integration, the area enclosed by the curves $y^2 = 4ax$ and $x^2 = 4ay$.
   Textbook Page No.: 4.41 (Jan 2010),(A/M 2011),(M/J 2013)

   Video Explanations
4. Find the area common to $y^2 = 4x$ and $x^2 = 4y$ using double integration.
   Textbook Page No.: 4.43 (N/D 2011),(N/D 2019)
5. Using double integral find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (M/J 2013),(N/D 2016)

   Textbook Page No.: 4.39
6. Evaluate $\int\int xy\ dxdy$ over the region in the positive quadrant bounded by $\frac{x}{a} + \frac{y}{b} = 1$.

   Textbook Page No.: 4.14 (A/M 2019)
7. Evaluate $\int\int xy\ dxdy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$.

   Textbook Page No.: 4.12 (Jan 2014), (Jan 2016),(M/J 2016),(N/D 2019)
8. Evaluate $\int\int (x - y)\ dxdy$ over the region between the line $y = x$ and the parabola $y = x^2$.

   Textbook Page No.: 4.9 Video Explanations (Jan 2011),(A/M 2017)
9. Find the area of the cardioid $r = a(1 + \cos\theta)$. (M/J 2014),(N/D 2014),(M/J 2015)

   Textbook Page No.: 4.45
10. Find the area inside the circle $r = a\sin\theta$ but lying outside the cardioids $r = a(1 - \cos\theta)$.

    Textbook Page No.: 4.49 (Jan 2009)
11. Find the area which is inside the circle $r = 3a\cos\theta$ and outside the cardioids $r = a(1 + \cos\theta)$. (Jan 2013)

Textbook Page No.: 4.48

## • Triple integrals and Volume of Solids

1. Evaluate $\int_0^a\int_0^b\int_0^c \left(x^2+y^2+z^2\right)dxdydz$. (A/M 2017)

   Textbook Page No.: 4.71

2. Evaluate $\int_0^{2a}\int_0^x\int_y^x (xyz)\, dzdydx$. (N/D 2019)

3. Evaluate $\int_0^{\log 2}\int_0^x\int_0^{x+y} e^{x+y+z}\, dxdydz$. (M/J 2009)

   Textbook Page No.: 4.77

4. Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$. (Jan 2012),(Jan 2013),(M/J 2015)

   Video Explanations

5. Evaluate $\int_0^a\int_0^{\sqrt{a^2-x^2}}\int_0^{\sqrt{a^2-x^2-y^2}} \frac{1}{\sqrt{a^2-x^2-y^2-z^2}}dzdydx$. (N/D 2011)

   Textbook Page No.: 4.74

6. Evaluate $\iiint xyz\,dx\,dy\,dz$ over the first octant of $x^2+y^2+z^2=a^2$.

   Video Explanation (A/M 2019),(Jan 2018)

7. Using triple integration, find the volume of the sphere $x^2+y^2+z^2=a^2$.

   Textbook Page No.: 4.80 (N/D 2010),(M/J 2015)

8. Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. (Jan 2010),(A/M 2011)

   Textbook Page No.: 4.81

9. Find the volume of the tetrahedron bounded by the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and the coordinate plane $x=0, y=0, z=0$. (M/J 2010),(N/D 2014)

Textbook Page No.: 4.83

10. Find the value of $\iiint xyz\ dxdydz$ over the first octant of $x^2+y^2+z^2=a^2$.
(A/M 2017),(Jan 2018)

11. Evaluate $\iiint x^2yz\ dxdydz$ taken over the tetrahedron bounded by the planes $x=0, y=0, z=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. (Jan 2011)

Textbook Page No.: 4.89

12. Evaluate $\iiint \frac{dzdydx}{(x+y+z+1)^3}$ where $V$ is the region bounded by $x=0, y=0,$ $z=0, x+y+z=1$. (N/D 2011),(Jan 2014),(Jan 2016),(M/J 2016)

Textbook Page No.: 4.88

# Unit – V (Differential Equations)

## • Differential Equations with Constant Coefficients

1. Solve $(D^2+2D+2)y=e^{-2x}+\cos 2x$. (N/D 2016)

   Textbook Page No.: 5.23 Video Explanations

2. Solve $(D^3+2D^2+D)y=e^{-x}+\cos 2x$. (Jan 2016)

   Textbook Page No.: 5.21

3. Solve $(D^2+16)y=\cos^3 x$. (Textbook Page No.: 5.20) (N/D 2010)

4. Solve : $(D^2+3D+2)y=\sin x+x^2$. (Textbook Page No.: 5.25) (M/J 2011)

5. Solve the equation $(D^2+5D+4)y=e^{-x}\sin 2x$. (A/M 2011),(ND 2012)

   Textbook Page No.: 5.28

6. Solve the equation $(D^2+4D+3)y=e^{-x}\sin x$. (M/J 2010)

   Video Explanations

7. Solve $(D^2-4D+3)y=e^{x}\cos 2x$. (M/J 2012)

Video Explanation

8. Solve $\left(D^2+4D+3\right)y=6e^{-2x}\sin x\sin 2x$. (N/D 2011)

Textbook Page No.: 5.29

9. Solve $\left(D^2-3D+2\right)y=xe^{3x}+\sin 2x$. (M/J 2015)

Video Explanation

10. Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=8xe^x\sin x$. (N/D 2013)

Video Explanation

11. Solve $\left(D^2+2D+1\right)y=xe^{-x}\cos x$. (Textbook Page No.: 5.33) (M/J 2016)

- ## Method of Variation of Parameters

1. Solve $\frac{d^2y}{dx^2}+a^2y=\tan ax$ by method of variation of parameters.

   Textbook Page No.: 5.73 (M/J 2009),(M/J 2011),(M/J 2014), (M/J 2016),(A/M 2019)

2. Apply method of variation of parameters to solve $\left(D^2+4\right)y=\cot 2x$.

   Textbook Page No.: 5.75 Video Explanation (N/D 2009),(N/D 2011),(Jan 2018)

3. Solve $\left(D^2+a^2\right)y=\sec ax$ using the method of variation of parameters.

   Textbook Page No.: 5.77 Video Explanations (M/ 2012),(N/D 2016)

4. Solve $\frac{d^2y}{dx^2}+y=\cos ecx$ by the method of variation of parameters.

   Textbook Page No.: 5.79 (A/M 2011),(ND 2012),(N/D 2019)

5. Solve $\left(D^2+1\right)y=\cos ecx\cot x$ using the method of variation of parameters.

   Textbook Page No.: 5.80 Video Explanations (A/M 2015)

6. Solve $\left(D^2+1\right)y=x\sin x$ by the method of variation of parameters. (M/J 2010)

   Textbook Page No.: 5.87

## • Euler's and Legendre's Equations

1. Solve $x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2 + \frac{1}{x^2}$. (M/J 2013)

   Textbook Page No.: 5.37

2. Solve $\left(x^2D^2 + 3xD + 4\right)y = x^2 + \cos\left(\log x\right)$. Video Explanation

3. Solve $\left(x^2D^2 - xD + 1\right)y = \sin(\log x)$. (N/D 2014)

   Video Explanation

4. Solve $\left(x^2D^2 - 2xD - 4\right)y = x^2 + 2\log x$. (M/J 2010)

   Textbook Page No.: 5.40

5. Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$. (N/D 2016)

   Video Explanation

6. Solve $\left(x^2D^2 - xD + 4\right)y = x^2\sin\left(\log x\right)$. (N/D 2009) ,(M/J 2012)

   Textbook Page No.: 5.42 Video Explanations

7. Solve $\left(x^2D^2 - 3xD + 4\right)y = x^2\cos\left(\log x\right)$. (N/D 2010)

   Video Explanation

8. Solve $\left(x^2D^2 - xD - 2\right)y = x^2\log x$. (M/J 2016)

   Textbook Page No.: 5.44

9. Solve $\left(x^2D^2 - xD + 1\right)y = \left(\frac{\log x}{x}\right)^2$. (Textbook Page No.: 5.46) (M/J 2014)

10. Solve the equation $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\log x}{x^2}$. (N/D 2012)

    Textbook Page No.: 5.48

11. Solve $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin\left[\log(1+x)\right]$. (A/M 2011)

Video Explanation

12. Solve $(1+x)^2\dfrac{d^2y}{dx^2}+(1+x)\dfrac{dy}{dx}+y=4\cos\left[\log(1+x)\right]$. (N/D 2011),(Jan 2018)

Textbook Page No.: 5.55

13. Solve $\left(x+2\right)^2\dfrac{d^2y}{dx^2}-\left(x+2\right)\dfrac{dy}{dx}+y=3x+4$. (A/M 2019),(N/D 2019)

Textbook Page No.: 5.54

14. Solve $\left(2x+7\right)^2 y''-6\left(2x+7\right)y'+8y=8x$. (Jan 2016)

Video Explanation

15. Solve $\left(3x+2\right)^2\dfrac{d^2y}{dx^2}+3\left(3x+2\right)\dfrac{dy}{dx}-36y=3x^2+4x+1$. (M/J 2013)

Textbook Page No.: 5.52

- ## Method of Undetermined Co-efficient

1. Solve $(D^2-6D+5)y = 3e^{2x}$ by method of undetermined coefficients.
   Textbook Page No.: 5.92
2. Solve $(D^2-5D+4)y = 2e^{x}$ by method of undetermined coefficients.
   Textbook Page No.: 5.93
3. Solve $(D^2+7D+12)y = \sin 2x$ by method of undetermined coefficients.
   Textbook Page No.: 5.94
4. Solve $(D^2+D-6)y = 3x^2$ by method of undetermined coefficients.
   Textbook Page No.: 5.97
5. Solve $(D^2-3D+2)y = e^{-x}+x^2$ by method of undetermined coefficients.
   Textbook Page No.: 5.98
6. Solve $(D^2+3D+2)y = 4e^{2x}+x$ by method of undetermined coefficients.(N/D 2019)
7. Solve $(D^2-5D+6)y = e^{3x}+\sin x$ by method of undetermined coefficients.
   Textbook Page No.: 5.99
8. Solve $\left(D^2-2D\right)y=5e^{x}\cos x$ by using method of undetermined coefficients. (Jan 2018)
   Textbook Page No.: 5.101
9. Solve $(D^2+2D+1)y = e^{x}\sin 2x$ by method of undetermined coefficients. (A/M 2019)
   Textbook Page No.: 5.102

## • System of Simultaneous Linear Differential Equations

1. Solve $Dx+y = \sin 2t$ and $-x+Dy = \cos 2t$. (Jan 2018),(N/D 2019)

   Video Explanation

2. Solve $\frac{dx}{dt}+2y=\sin 2t,\ \frac{dy}{dt}-2x=\cos 2t$. (N/D 2009),(M/J 2012)

   Textbook Page No.: 5.59 Video Explanations

3. Solve $\frac{dx}{dt}+2y=-\sin t,\ \frac{dy}{dt}-2x=\cos t$ given $x=1$, $y=0$ at $t=0$. (N/D 2010)

   Textbook Page No.: 5.61

4. Solve $\frac{dx}{dt}-y=t$ and $\frac{dy}{dt}+x=t^2$. (A/M 2011),(M/J 2016)

   Textbook Page No.: 5.63

5. Solve $\frac{dx}{dt}+y=e^t,\ x-\frac{dy}{dt}=t$. (N/D 2012),(N/D 2014)

   Textbook Page No.: 5.65

6. Solve $\frac{dx}{dt}+2x-3y=t$ and $\frac{dy}{dt}-3x+2y=e^{2t}$. (N/D 2011)

   Textbook Page No.: 5.67

7. Solve the simultaneous differential equations: $\frac{dx}{dt}+\frac{dy}{dt}+3x=\sin t,\ \frac{dx}{dt}+y-x=\cos t$.

   Textbook Page No.: 5.69 (M/J 2015),(A/M 2019)

# Textbook for Reference:

**"ENGINEERING MATHEMATICS - I"**

**Publication: Sri Hariganesh Publications** **Author: C. Ganesan**

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----All the Best----