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## Unit – I (Testing of Hypothesis)

### • Student's 't' test

1. A random sample of 10 boys has the following IQ's 70, 83, 88, 95, 98, 100, 101, 107, 110 and 120. Do these data support the assumption of a population mean IQ of 100 at 5% level of significance? (N/D 2017),(N/D 2018)

Textbook Page No.: 1.2

2. The heights of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inches. Is it reasonable to believe that the average height is greater than 64 inches? (A/M 2011)

Textbook Page No.: 1.3

3. A sample of 10 boys had the I.Q's: 70, 120, 110, 101, 88, 83, 95, 98, 100 and 107. Test whether the population mean I.Q may be 100. (N/D 2012)

Textbook Page No.: 1.2

4. The IQ's of 10 girls are respectively 120, 110, 70, 88, 101, 100, 83, 98, 95, 107. Test whether these two proportions are same. (M/J 2016)

Textbook Page No.: 1.2

5. The height of six randomly chosen sailors are (in inches): 63, 65, 68, 69, 71 and 72. Those of 10 randomly chosen soldiers are 61, 62, 65, 66, 69, 69, 70, 71, 72 and 73.

Discuss, the height that these data thrown on the suggestion that sailors are on the average taller than soldiers ( $t_{0.05}(14) = 1.76$ ). (N/D 2014)

Textbook Page No.: 1.2

### • F – test

1. The nicotine content in milligram of 2 samples of tobacco were found to be as follows:

Sample A	24	27	26	21	25	
Sample B	27	30	28	31	22	36

(A/M 2018)

Can it be said that these sample were from normal population with the same mean.

Textbook Page No.: 1.7

2. Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables. (M/J 2014)

Sample I: 18 13 12 15 12 14 16 14 15  
 Sample II: 16 19 13 16 18 13 15

Do the estimates of population variance differ significantly at 5% level of significance?

Textbook Page No.: 1.8

3. Time taken by workers in performing a job are given below:

Type I 21 17 27 28 24 23 -  
 Type II 28 34 43 36 33 35 39

Test whether there is any significant difference between the variances of time distribution. (N/D 2013)

Textbook Page No.: 1.10

4. Test whether there is any significant difference between the variances of the populations from which the following samples are taken: (N/D 2012), (N/D 2017)

Sample I: 20 16 26 27 23 22  
 Sample II: 27 33 42 35 32 34 38

Textbook Page No.: 1.10

5. Test if the difference in the means is significant for the following data: (N/D 2010)

Sample I:	76	68	70	43	94	68	33	
Sample II:	40	48	92	85	70	76	68	22

Textbook Page No.: 1.10

6. Two random samples gave the following results:

Sample	Size	Sample mean	Sum of squares of deviation from the mean
1	10	15	90
2	12	14	108

Test whether the samples have come from the same normal population. (M/J 2012)

Textbook Page No.: 1.10

### • Chi-Square test (Goodness of fit)

1. Theory predicts that the proportion of beans in four groups A, B, C, D should be 9:3:3:1. In an experiment among 1600 beans, the numbers in the four groups were 882, 313, 287 and 118. Does the experiment support the theory? (M/J 2012), (M/J 2016)

Textbook Page No.: 1.16

2. Five coins are tossed 320 times. The number of heads observed is given below:

Number of Heads	0	1	2	3	4	5
Frequency	15	45	85	95	60	20

Examine whether the coin is unbiased. Use 5% level of significance. (A/M 2018)

Textbook Page No.: 1.16

3. 4 coins were tossed 160 times and the following results were obtained:

No. of heads:	0	1	2	3	4
Observed frequencies:	17	52	54	31	6

Under the assumption that the coins are unbiased, find the expected frequencies of getting 0, 1, 2, 3, 4 heads and test the goodness of fit. (A/M 2011)

Textbook Page No.: 1.14

4. The following data gives the number of aircraft accidents that occurred during the various days of a week. Find whether the accidents are uniformly distributed over the week. (N/D 2010)

Days:	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents:	14	16	8	12	11	9	14

Textbook Page No.: 1.12

5. The demand for a particular spare part in a factory was found to vary from day-to-day. In a sample study the following information was obtained.

Days:	Mon	Tues	Wed	Thurs	Fri	Sat
No. of spare parts demanded:	1124	1125	1110	1120	1126	1115

Test the hypothesis that the number of parts demanded does not depend on the day of the week. ( $\chi^2_{0.05}(5) = 11.07$ ) (N/D 2014)

Textbook Page No.: 1.13

### • Chi-Square test (Independence of attributes)

1. Mechanical engineers testing a new arc welding technique, classified welds both with respect to appearance and an X-ray inspection. (A/M 2018)

		Appearance		
		Bad	Normal	Good
X-ray	Bad	20	7	3
	Normal	13	51	16
	Good	7	12	21

Test for independence using 0.05 level of significance.

Textbook Page No.: 1.24

2. Using the data given in the following table to test at 1% level of significance whether a person's ability in Mathematics is independent of his/her interest in Statistics. (N/D 2017), (N/D 2018)

		Ability in Mathematics		
		Low	Average	High
Interest in Statistics	Low	63	42	15
	Average	58	61	31
	High	14	47	29

Textbook Page No.: 1.22

3. Out of 8000 graduates in a town 800 are females, out of 1600 graduate employees 120 are females. Use  $\chi^2$  to determine if any distinction is made in appointment on the basis of sex. Value of  $\chi^2$  at 5% level for one degree of freedom is 3.84. (A/M 2010)

Textbook Page No.: 1.18

4. An automobile company gives you the following information about age groups and the liking for particular model of car which it plans to introduce. On the basis of this data can it be concluded that the model appeal is independent of the age group.

$$(\chi^2_{0.05}(3) = 7.815)$$

(A/M 2010)

Persons who:	Below 20	20 – 39	40 – 59	60 and above
Liked the car:	140	80	40	20
Disliked the car:	60	50	30	80

Textbook Page No.: 1.20

5. Test of the fidelity and selectivity of 190 radio receivers produced the results shown in the following table:

Selectivity	Fidelity		
	Low	Average	High
Low	6	12	32
Average	33	61	18
High	13	15	0

Use the 0.01 level of significance to test whether there is a relationship between fidelity and selectivity.

(A/M 2011)

Textbook Page No.: 1.24

### • Large Sample (n > 30)

#### Single Mean & Difference of Means:

1. A sample of 900 members has a mean 3.4 cm and standard deviation 2.61 cm. Is the sample from a large population of mean 3.25 cms and standard deviation of 2.61cms? (Test at 5% level of significance. The value of  $z$  at 5% level is  $|z_{\alpha}| < 1.96$ ). (A/M 2010)

Textbook Page No.: 1.25

2. The means of two large samples of 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same populations of standard deviation 2.5 inches? (M/J 2012), (N/D 2018)

Textbook Page No.: 1.26

3. A mathematics test was given to 50 girls and 75 boys. The girls made an average grade of 76 with a SD of 6, while boys made an average grade of 82 with a SD of 2. Test whether there is any significant difference between the performance of boys and girls.

Textbook Page No.: 1.27

(N/D 2012),(M/J 2016)

4. A random sample of 100 bulbs from a company P shows a mean life 1300 hours and standard deviation of 82 hours. Another random sample of 100 bulbs from company Q showed a mean life 1248 hours and standard deviation of 93 hours. Are the bulbs of company P superior to bulbs of company Q at 5% level of significance? (N/D 2017)

Textbook Page No.: 1.28

5. The sales manager of a large company conducted a sample survey in two places A and B taking 200 samples in each case. The results were the following table. Test whether the average sales is the same in the 2 areas at 5% level. (N/D 2013)

	Place A	Place B
Average Sales	Rs. 2,000	Rs. 1,700
S.D	Rs. 200	Rs. 450

Textbook Page No.: 1.30

6. Examine whether the difference in the variability in yields is significant at 5% level of significance, for the following. (N/D 2010)

	Set of 40 plots	Set of 60 plots
Mean yield per plot	1256	1243
S.D. per plot	34	28

Textbook Page No.: 1.31

### Single Proportion & Difference of Proportions:

7. 20 people were attacked by a disease and only 18 survived. Will you reject the hypothesis that the survival rate, if attacked by this disease is 85% is favor of the hypothesis that is more at 5% level? (N/D 2013)

Textbook Page No.: 1.32

8. A Manufacturer of light bulbs claims that an average of 2% of the bulbs manufactured by him are defective. A random sample of 400 bulbs contained 13 defective bulbs. On the basis of the sample, can you support the manufacturer's claim at 5% level of significance? (M/J 2014)

Textbook Page No.: 1.33

9. 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test whether these two proportions are same. (M/J 2016)

Textbook Page No.: 1.34

10. In a random sample of 100 men taken from village A, 60 were found to be consuming alcohol. In another sample of 200 men taken from village B, 100 were found to be consuming alcohol. Do the two villages differ significantly in respect of the proportion of men who consume alcohol? (M/J 2014)

Textbook Page No.: 1.35

11. Before an increase in excise duty on tea, 800 persons out of a sample of 1000 persons were found to be tea drinkers. After an increase in duty, 800 people were tea drinkers in a sample of 1200 people. Using standard error of proportion, state whether there is a significant decrease in the consumption of tea after the increase in excise duty. ( $z_{\alpha}$  at 5% level 1.645, 1% level 2.33). (A/M 2010)

Textbook Page No.: 1.36

12. A machine puts out 16 imperfect articles in a sample of 500. After it was overhauled, it puts out 3 imperfect articles in a sample of 100. Has the machine improved in its performance? (N/D 2010)(N/D2012)

Textbook Page No.: 1.37

13. Before an increase in excise duty on tea, 900 persons out of a sample of 1100 persons were found to be tea drinkers. After an increase in excise duty, 900 person were tea drinkers in a sample of 1300. Using standard error of proportion, state whether there is a significant decrease in the consumption of tea after the increase in excise duty?

Textbook Page No.: 1.37

(N/D 2013)

14. In a random sample of 1000 people from city A, 400 are found to be consumers of wheat. In a sample of 800 from city B, 400 are found to be consumers of wheat. Does this data give a significant difference between the two cities as far as the proportion of wheat consumers is concerned? (A/M 2011)

Textbook Page No.: 1.38

## Unit – II (Design of Experiments)

### • Completely Randomized Design (One – Way Classification)

- The following are the number of mistakes made in 5 successive days by 4 technicians working for a photographic laboratory test at a level of significance  $\alpha = 0.01$ . Test whether the difference among the four sample means can be attributed to chance. (A/M 2011)

Technician			
I	II	III	IV
6	14	10	9
14	9	12	12
10	12	7	8
8	10	15	10
11	14	11	11

Textbook Page No.: 2.2

### • Randomized Block Design (Two – Way Classification)

- A chemist wishes to test the effect of four chemical agents on the strength of a particular type of cloth. Because there might be variability from one bolt to another, the chemist decides to use a randomized block design, with the bolts of cloth consider as blocks, she selects five bolts and applies all four chemicals in random order to each bolt. The resulting tensile strengths follows: (A/M 2018)

Chemical	Bolt				
	1	2	3	4	5
1	73	68	74	71	67
2	73	67	75	72	70
3	75	68	78	73	68
4	73	71	75	75	69

Does the tensile strength depend on chemical? Test at  $\alpha = 0.10$ .

Textbook Page No.: 2.31



2. Three varieties of coal were analysed by 4 chemists and the ash content is tabulated here. Perform an analysis of variance. (M/J 2016)

		Chemists			
		A	B	C	D
Coal	I	8	5	5	7
	II	7	6	4	4
	III	3	6	5	4

Textbook Page No.: 2.31

3. Analyse the following RBD and find your conclusion. (N/D 2013)

		Treatments			
		$T_1$	$T_2$	$T_3$	$T_4$
Blocks	$B_1$	12	14	20	22
	$B_2$	17	27	19	15
	$B_3$	15	14	17	12
	$B_4$	18	16	22	12
	$B_5$	19	15	20	14

Textbook Page No.: 2.15

4. A set of data involving four “four tropical feed stuffs A, B, C, D” tried on 20 chicks is given below. All the twenty chicks are treated alike in all respects except the feeding treatments and each feeding treatment is given to 5 chicks. Analyze the data. Weight gain of baby chicks fed on different feeding materials composed of tropical feed stuffs. (A/M 2010)

						Total
A	55	49	42	21	52	219
B	61	112	30	89	63	355
C	42	97	81	95	92	407
D	169	137	169	85	154	714
Grand Total						G = 1695

Textbook Page No.: 2.18

5. Four varieties A, B, C, D of a fertilizer are tested in a RBD with 4 replications. The plot yields in pounds are as follows:

A12	D20	C16	B10
D18	A14	B11	C14
B12	C15	D19	A13
C16	B11	A15	D20

Analyse the experimental yield.

(M/J 2012),(M/J 2014)

Textbook Page No.: 2.21

6. The result of an RBD experiment on 3 blocks with 4 treatments A, B, C, D are tabulated here. Carry out an analysis of variance. (M/J 2016)

	Treatment effects			
I	A36	D35	C21	B36
II	D32	B29	A28	C31
III	B28	C29	D29	A26

Textbook Page No.: 2.24

7. Carry out ANOVA (Analysis of variance) for the following. (N/D 2010)

	A	B	C	D	
Workers	1	44	38	47	36
	2	46	40	52	43
	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

Textbook Page No.: 2.27

8. The following data represent the number of units of production per day turned out by 5 different workers using 4 different types of machines. (A/M 2011),(M/J 2013)

	Machine Type				
	A	B	C	D	
Workers	1	44	38	47	36
	2	46	40	52	43
	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

(i) Test whether the mean production is the same for the different machine types.

(ii) Test whether the 5 men differ with mean productivity.

Textbook Page No.: 2.27

9. The sales of 4 salesmen in 3 seasons are tabulated here. Carry out an analysis of variance. (N/D 2012)

	Salesmen			
Seasons	A	B	C	D
Summer	36	36	21	35

Winter 28 29 31 32

Monsoon 26 28 29 29

### • Latin Square (Three – Way Classification)

1. A Latin square design was used to compare the bond strengths of gold semi conductor lead wires bounded to the lead terminal by 5 different methods, A, B, C, D and E. The bonds were made by 5 different operators and the devices were encapsulated using 5 different plastics. With the following results, expressed as pounds of force required to break the bond. (A/M 2018)

		Operator				
		1	2	3	4	5
Plastics	1	A 3	B 2.4	C 1.9	D 2.2	E 1.7
	2	B 2.1	C 2.7	D 2.3	E 2.5	A 3.1
	3	C 2.1	D 2.6	E 2.5	A 2.9	B 2.1
	4	D 2.0	E 2.5	D 3.2	B 2.5	C 2.2
	5	E 2.1	A 3.6	B 2.4	C 2.4	D 2.1

Analyse these results and test with 0.01 level of significance.

Textbook Page No.: 2.51

2. A variable trial was conducted on wheat with 4 varieties in a Latin Square design. The plan of the experiment and per plot yield are given below:

C25 B23 A20 D20  
A19 D19 C21 B18  
B19 A14 D17 C20  
D17 C20 B21 A15

Analyse the data.

(M/J 2012)

Textbook Page No.: 2.38

3. A farmer wishes to test the effect of 4 fertilizers A, B, C, D on the yield of wheat. The fertilizers are used in a LSD and the result are tabulated here. Perform an analysis of variance. (N/D 2012)

A18 C21 D25 B11  
D22 B12 A15 C19  
B15 A20 C23 D24  
C22 D21 B10 A17

Textbook Page No.: 2.33

4. Analyse the following of Latin square experiment. (M/J 2013)

A12	D20	C16	B10
D18	A14	B11	C14
B12	C15	D19	A13
C16	B11	A15	D20

Textbook Page No.: 2.42

5. The following is a Latin square of a design when 4 varieties of seed are being tested. Set up the analysis of variance table and state your conclusion. You can carry out the suitable change of origin and scale. (N/D 2013)

A 110	B 100	C 130	D 120
C 120	D 130	A 110	B 110
D 120	C 100	B 110	A 120
B 100	A 140	D 100	C 120

Textbook Page No.: 2.51

6. Analyse the variance in the Latin square of yields (in kgs) of paddy where P, Q, R, S denote the different methods of cultivation: (M/J 2014)

S122	P121	R123	Q122
Q124	R123	P122	S125
P120	Q119	S120	R121
R122	S123	Q121	P122

Examine whether different method of cultivation have significantly different yields.

Textbook Page No.: 2.51

7. In a Latin square experiment given below are the yields in quintals per acre on the paddy crop carried out for testing the effect of five fertilizers A, B, C, D, E. Analyze the data for variations. (A/M 2011)

B 25	A 18	E 27	D 30	C 27
A 19	D 31	C 29	E 26	B 23

C 28   B 22   D 33   A 18   E 27

E 28   C 26   A 20   B 25   D 33

D 32   E 25   B 23   C 28   A 20

Textbook Page No.: 2.46

8. The following is a Latin square of a design when 4 varieties of seeds are being tested. Set up the analysis of variance table and state your conclusion. You may carry out suitable change of origin and scale. (M/ J 2013)

A 105   B 95   C 125   D 115

C 115   D 125   A 105   B 105

D 115   C 95   B 105   A 115

B 95   A 135   D 95   C 115

Textbook Page No.: 2.52

9. A company wants to produce cars for its own use. It has to select the make of the car out of the four makes A, B, C and D available in the market. For this he tries four cars of each make by assigning the cars to four drivers to run on four different routes. The efficiency of cars is measured in terms of time in hours. The layout and time consumed is as given below.

Routes	Drivers			
	1	2	3	4
1	18 (C)	12 (D)	16 (A)	20 (B)
2	26 (D)	34 (A)	25 (B)	31 (C)
3	15 (B)	22 (C)	10 (D)	28 (A)
4	30 (A)	20 (B)	15 (C)	9 (D)

Analyse the experimental data and draw conclusions. ( $F_{0.05}(3,5) = 5.41$ ) (N/D 2014)

Textbook Page No.: 2.52

**Unit – III (Solution of Equations and Eigenvalue Problems)****• Newton – Raphson method**

1. Using Newton-Raphson method, solve  $x \log_{10} x = 12.34$  taking the initial value  $x_0$  as 10. (M/J 2012)

(Or)

Solve the equation  $x \log_{10} x = 1.2$  using Newton-Raphson method. (M/J 2014)

Textbook Page No.: 3.11

2. Find the real positive root for the equation  $3x - \cos x = 1$  by Newton-Raphson method correct to 6 decimal places. (A/M 2011),(N/D 2013),(N/D 2017)

Textbook Page No.: 3.9

**• Gauss – Jacobi method & Gauss – Seidel method**

1. Solve, by Gauss-Jacobi method, the system of following equations correct to three decimal places  $27x + 6y - z = 85$ ;  $6x + 15y + 2z = 72$ ;  $x + y + 54z = 110$ .

Textbook Page No.: 3.25 (N/D 2018)

2. Using Gauss-Seidel method, solve the following system of linear equations  $x + 5y - z = 10$ ;  $4x + 2y + z = 14$ ;  $x + y + 8z = 20$ .

Textbook Page No.: 3.29

3. Apply Gauss-Seidel method to solve the equations  $20x + y - 2z = 17$ ;  $3x + 20y - z = -18$ ;  $2x - 3y + 20z = 25$ .

Textbook Page No.: 3.31

4. Solve the system of equations using Gauss-Seidal method  $x + y + z = 9$ ;  $2x - 3y + 4z = 13$ ;  $3x + 4y + 5z = 40$ . (N/D 2012)

5. Find the solution, to three decimals, of the system using Gauss-Seidal method  $8x + 11y - 4z = 95$ ;  $7x + 52y + 13z = 104$ ;  $3x + 8y + 29z = 71$ . (N/D 2014)

6. Solve the following set of equations using Gauss-Seidal iterative procedure  $-10x + 2y + 2z = 4$ ;  $x - 10y + 2z = 18$ ;  $x + y - 10z = 45$ . (M/J 2014)

7. Solve the following system of equations using Gauss-Seidal iterative method  $27x + 6y - z = 85$ ,  $6x + 15y + 2z = 72$ ,  $x + y + 54z = 110$ . (M/J 2012)

8. Solve the following equations by Gauss-Seidal method  
 $x + y + 54z = 110$ ,  $27x + 6y - z = 85$ ,  $6x + 15y + 2z = 72$ .  
(A/M 2011),(N/D 2012),(N/D 2017), (A/M 2018)
9. Solve  $5x - y + z = 10$ ;  $2x + 4y = 12$  and  $x + y + 5z = -1$  using Gauss Seidel method.  
(A/M 2010)
10. Solve by Gauss-Seidel method  $6x + 3y + 12z = 35$ ;  $8x - 3y + 2z = 20$ ;  
 $4x + 11y - z = 33$ .  
(N/D 2010)
11. Solve by Gauss-Seidal method  $28x + 4y - z = 32$ ;  $x + 3y + 10z = 24$ ;  
 $2x + 17y + 4z = 35$ .  
(M/J 2013)
12. Solve by Gauss-Seidal method  $x + y + 9z = 15$ ;  $x + 17y - 2z = 48$ ;  
 $30x - 2y + 3z = 75$ .  
(M/J 2016)

### • Gauss Elimination method & Gauss Jordan method

1. Solve the system of equations using Gauss-elimination method  
 $5x - 2y + z = 4$ ,  $7x + y - 5z = 8$ ,  $3x + 7y + 4z = 10$ .  
(N/D 2014)  
Textbook Page No.: 3.17
2. Solve the following equations by Gauss elimination method  
 $x + y + z = 9$ ,  $2x - 3y + 4z = 13$ ,  $3x + 4y + 5z = 40$ .  
(N/D 2012)  
Textbook Page No.: 3.15
3. Solve the following system of equation by Gauss elimination method:  
 $2x + y + z = 10$ ;  $3x + 2y + 3z = 18$ ;  $x + 4y + 9z = 16$ .  
(A/M 2010),(N/D 2018)  
Textbook Page No.: 3.18
4. Solve by Gauss-Elimination method  $2x + 3y - z = 5$ ;  $4x + 4y - 3z = 3$ ;  
 $2x - 3y + 2z = 2$ .  
(M/J 2009)  
Textbook Page No.: 3.16
5. Solve by Gauss-Elimination method  $3x + 4y + 5z = 18$ ;  $2x - y + 8z = 13$ ;  
 $5x - 2y + 7z = 20$ .  
(M/J 2013)
6. Using Gauss-Jordan method to solve  
 $2x - y + 3z = 8$ ,  $-x + 2y + z = 4$ ,  $3x + y - 4z = 0$ .  
(N/D 2015)  
Textbook Page No.: 3.21

7. Using Gauss-Jordan, solve the following system  $10x + y + z = 12$ ;  $2x + 10y + z = 13$ ;  
 $x + y + 5z = 7$ . (N/D 2010)

Textbook Page No.: 3.20

8. Solve the following equations using Gauss-elimination method  
 $5x - y = 9$ ,  $-x + 5y - z = 4$ ,  $-y + 5z = -6$ .

Textbook Page No.: 3.23

9. Solve the following system of equation by Gauss-Jordan  $2x + y + 4z = 12$ ;  
 $8x - 3y + 2z = 20$ ;  $4x + 11y - z = 33$ . (A/M 2018)

10. Solve the following equations using Gauss-elimination method  
 $2x + y + 4z = 12$ ,  $8x - 3y + 2z = 20$ ,  $4x + 11y - z = 33$ . (M/J 2016)

11. Solve the following system of equations by Gauss-Elimination method  $2y - 3z = -5$ ;  
 $x + 4y - 7z + t = -8$ ;  $2x - y - t = -4$ ;  $x + y + z = 6$ ;. (A/M 2018)

12. Solve the system of equations by Gauss-Elimination method  $x_1 + x_2 + x_3 + x_4 = 2$ ;  
 $2x_1 - x_2 + 2x_3 - x_4 = -5$ ;  $3x_1 + 2x_2 + 3x_3 + 4x_4 = 7$ ;  $x_1 - 2x_2 - 3x_3 + 2x_4 = 5$ .  
(N/D 2013)

13. Solve the system of equations by Gauss-Jordan method  $x + y + z + w = 1$ ;  
 $2x - y + 2z - w = -5$ ;  $3x + 2y + 3z + 4w = 7$ ;  $x - 2y - 3z + 2w = 5$ . (M/J 2013)

### • Matrix inversion by Gauss Jordan method

1. Using Gauss Jordan method, find the inverse  $A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$ . (A/M 2018)

Textbook Page No.: 3.37

2. Using Gauss Jordon method, find the inverse of  $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ .  
(A/M 2010), (M/J 2012)

Textbook Page No.: 3.40

3. If  $A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ , find  $A^{-1}$  by Gauss-Jordan method. (N/D 2012)

Textbook Page No.: 3.42



4. By Gauss-Jordan method, find the inverse of  $A = \begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}$ .

Textbook Page No.: 3.35

(A/M 2011), (M/J 2016), (N/D 2017)

5. Find the inverse of the matrix  $\begin{pmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{pmatrix}$  by Gauss-Jordan method. (N/D 2013)

6. Find the inverse of the matrix  $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 2 & 2 \end{pmatrix}$  using Gauss-Jordan method.

(N/D 2014)

Textbook Page No.: 3.44

7. Find the inverse of  $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 3 & -1 & -4 \end{pmatrix}$  using Gauss-Jordan method. (N/D 2010)

Textbook Page No.: 3.44

8. By Gauss Jordan elimination method, find the inverse of the matrix  $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & -1 \\ 2 & -1 & 2 \end{pmatrix}$ .

(M/J 2014)

### • Eigen values of a matrix by Power method

1. Using the power method, find the largest eigenvalue and the corresponding eigenvector for the matrix  $A = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$ . Let  $X_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ . (A/M 2018)

Textbook Page No.: 3.44

2. Find the dominant eigenvalue of  $\begin{pmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{pmatrix}$  by power method. (N/D 2012)

Textbook Page No.: 3.50

3. Find all the eigenvalue of  $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$  using power method. Using

 $x_1 = (1, 0, 0)^T$  as initial vector.

(N/D 2010)

Textbook Page No.: 3.47

4. Find the numerically largest eigenvalue of  $A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$  and the corresponding eigenvector.  
(A/M 2013), (N/D 2017), (M/J 2016)

Textbook Page No.: 3.45

5. Find the dominant eigenvalue and its eigenvector of the matrix by power method

$$A = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}. \quad (\text{N/D 2014})$$

Textbook Page No.: 3.52

6. Using power method, find all the eigenvalues of  $A = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$ . (M/J 2013)

Textbook Page No.: 3.52

7. Determine the largest eigenvalue and the corresponding eigenvector of the matrix

$$A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \text{ with the initial vector } X^{(0)} = [1, 1, 1]^T. \quad (\text{A/M 2010})$$

Textbook Page No.: 3.52

8. Find the numerically largest eigenvalue of  $A = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}$  by power method.  
(M/J 2012), (M/J 2014)

9. Find the largest eigenvalue of the matrix  $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 0 \end{pmatrix}$  by power method. Also find its corresponding eigenvector.  
(N/D 2013)

**Unit – IV (Interpolation, Numerical Differential & Integration)****• Lagrange's & Newton's divided difference interpolation**

1. Using Lagrange's interpolation formula, find the polynomial  $f(x)$  from the following data:

(N/D 2017)

$$x: 0 \quad 1 \quad 4 \quad 5$$

$$f(x): 4 \quad 3 \quad 24 \quad 39$$

2. Using Lagrange's interpolation formula, find  $y(10)$  from the following table. (A/M 2011)

$$x: 5 \quad 6 \quad 9 \quad 11$$

$$y: 12 \quad 13 \quad 14 \quad 16$$

Textbook Page No.: 4.4

3. Using Lagrange's interpolation, find the value of  $f(3)$ , from the following table:

(M/J 2012)

$$x: 0 \quad 1 \quad 2 \quad 5$$

$$f(x): 2 \quad 3 \quad 12 \quad 147$$

Textbook Page No.: 4.5

4. Using Lagrange's method, find the value of  $f(3)$  from the following table: (N/D 2013)

$$x: 0 \quad 1 \quad 2 \quad 3$$

$$f(x): 2 \quad 3 \quad 12 \quad 147$$

Textbook Page No.: 4.5

5. Use Lagrange's formula to fit a polynomial to the following data hence find  $y(x = 1)$ .

(N/D 2010)

$$x: -1 \quad 0 \quad 2 \quad 3$$

$$y: -8 \quad 3 \quad 1 \quad 12$$

6. Find polynomial  $f(x)$  by using Lagrange's formula and hence find  $f(4)$  for

(M/J 2014)

$x:$	1	3	5	7
$f(x):$	24	120	336	720

7. Given the table of values

$x :$	50	52	54	56
$\sqrt[3]{x} :$	3.684	3.732	3.779	3.825

Use Lagrange's formula to find  $\sqrt[3]{53}$ . (N/D 2014)

8. From the following values, find  $f(x)$  and hence find  $f(6)$  by Newton's divided difference formula. (N/D 2017)

$x:$	1	2	7	8
$f(x):$	1	5	5	4

Textbook Page No.: 4.17

9. Use Newton divided difference formula to calculate  $f(3)$ ,  $f'(3)$  and  $f''(3)$  from the following table: (A/M 2010)

$x :$	0	1	2	4	5	6
$f(x) :$	1	14	15	5	6	19

10. Using Newton's divided difference formula, find the values of  $f(2)$ ,  $f(8)$  and  $f(15)$  given the following table. (A/M 2011), (M/J 2013), (N/D 2013)

$x :$	4	5	7	10	11	13
$f(x) :$	48	100	294	900	1210	2028

Textbook Page No.: 4.14

11. Given the set of tabulated points  $(1, -3)$ ,  $(3, 9)$ ,  $(4, 30)$  and  $(6, 132)$  obtain the value of  $y$  when  $x = 2$  using Newton's divided difference formula. (N/D 2014)

Textbook Page No.: 4.10

12. Find the cubic polynomial  $y(x)$  for (N/D 2012)

$x :$	-1	0	2	3
$y(x) :$	-8	3	1	12

### • Newton's forward and backward difference interpolation

1. Interpolate  $y(12)$ , if (N/J 2016)

$x$	10	15	20	25	30	35
$y(x)$	35	33	29	27	22	14

2. Using Newton's forward interpolation formula, find the polynomial  $f(x)$  satisfying the following data. Hence evaluate  $f(x)$  at  $x = 5$ . (M/J 2012)

$$\begin{array}{l} x: \quad 4 \quad 6 \quad 8 \quad 10 \\ f(x): \quad 1 \quad 3 \quad 8 \quad 16 \end{array}$$

Textbook Page No.: 4.20

3. Construct Newton's forward interpolation polynomial for the following data:

$x:$	1	2	3	4	5
$f(x):$	1	-1	1	-1	1

and hence find  $f(3.5)$ ,  $f'(3.5)$ . (M/J 2014)

4. Find  $y(1976)$  from the following (N/D 2010)

$x:$	1941	1951	1961	1971	1981	1991
$y:$	20	24	29	36	46	51

5. From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 46 and 63. (A/M 2011)

Age $x:$	45	50	55	60	65
Premium $y:$	114.84	96.16	83.32	74.48	68.48

6. Find  $y(22)$ , given that (N/D 2012)

$x:$	20	25	30	35	40	45
$y(x):$	354	332	291	260	231	204

### • Approximation of derivatives using interpolation

1. Find the first derivative of  $f(x)$  at  $x = 0.4$  from the following table: (A/M 2018)

$x$	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

2. Find the first two derivatives of  $x^{1/3}$  at  $x = 56$ , given the table below: (A/M 2018)

$x$	50	51	52	53	54	55	56
$y = x^{1/3}$	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

3. Compute  $f'(0)$  and  $f''(4)$  from the following data: (N/D 2012)

$x:$	0	1	2	3	4
$f(x):$	1	2.718	7.381	20.086	54.598

4. From the following table of values of  $x$  and  $y$ , obtain  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  for  $x = 1.2$ . (A/M 2010)

$x:$	1.0	1.2	1.4	1.6	1.8	2.0	2.2
$y:$	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

5. Find the first and second derivatives of  $f(x)$  at  $x = 1.5$  if (N/D 2014)

$x:$	1.5	2.0	2.5	3.0	3.5	4.0
$f(x):$	3.375	7.000	13.625	24.000	38.875	59.000

Textbook Page No.: 4.34

6. The population of a certain town is given below. Find the rate of growth of the population in 1931, 1941, 1961 and 1971. (M/J 2013)

Year $x:$	1931	1941	1951	1961	1971
Population in thousands $y:$	40.62	60.80	79.95	103.56	132.65

7. Find the value of  $\cos(1.74)$ , using suitable formula from the given data. (N/D 2017)

$x:$	1.7	1.74	1.78	1.82	1.86
$\sin x:$	0.9916	0.9857	0.9781	0.9691	0.9584

8. Find  $y'(1)$ , if (M/J 2016)

$x:$	-1	0	2	3
$y(x):$	-8	3	1	12

9. Find  $y'(1)$ , if (N/D 2012)

$x:$	0	2	3	4	7	9
$y(x):$	4	26	58	112	466	922

**• Numerical integration using Trapezoidal & Simpson's Rule**

1. Evaluate  $\int_{-1}^1 \frac{dx}{1+x^2}$  by Trapezoidal rule, dividing the range into eight equal parts.

Textbook Page No.: 4.42

(M/J 2016)

2. Evaluate  $\int_0^2 e^x dx$  by using Trapezoidal rule taking 6 sub intervals. (A/M 2018)

3. Evaluate  $\int_0^1 \frac{dx}{1+x}$  by using Simpson's one-third rule and hence deduce the value of  $\log_e 2$ . (M/J 2014)

4. Evaluate  $\int_0^{\pi/2} \sin x dx$ , by trapezoidal and Simpson's  $\left(\frac{1}{3}\right)$  rules by dividing the range into 10 equal parts. Verify your answer with integration. (N/D 2012),(M/J 2013)

Textbook Page No.: 4.45

5. Taking  $h = \frac{\pi}{5}$ , evaluate  $\int_0^{2\pi} \sin x dx$  by Simpson's 1/3 rule. Verify the answer with integration. (N/D 2010)

Textbook Page No.: 4.44

6. Evaluate  $\int_0^2 \frac{dx}{x^2+x+1}$  to three decimals, dividing the range of integration into 8 equal parts using Simpson's rule. (M/J 2012)

7. Evaluate  $\int_0^1 \frac{dx}{1+x^2}$  by Simpson's (1/3) rule, dividing the range into four equal parts. (M/J 2016)

8. Evaluate  $\int_0^6 \frac{1}{1+x} dx$  using Trapezoidal rule. Verify the answer with direct integration.

Textbook Page No.: 4.47

(N/D 2010)

9. The table below gives the velocity  $V$  of a moving particle at time  $t$  seconds. Find the distance covered by the particle in 12 seconds and also the acceleration at  $t = 2$  seconds, using Simpson's rule. (A/M 2011)

$t$ :	0	2	4	6	8	10	12
$V$ :	4	6	16	34	60	94	136

10. The velocity  $v$  of a particle at a distance  $s$  from a point on its path is given as follows:

$s$ in meter:	0	10	20	30	40	50	60
$v$ m/sec:	47	58	64	65	61	52	38

Estimate the time taken to travel 60 meters by using Trapezoidal rule and Simpson's rule. (M/J 2014)

Textbook Page No.: 4.51

11. The velocities of a car (running on a straight road) at intervals of 2 minutes are given below. (N/D 2014)

Time in minutes:	0	2	4	6	8	10	12
Velocities in km/hr:	0	22	30	27	18	7	0

Apply Simpson's rule to find the distance covered by the car.

12. A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is in the table below. Using trapezoidal rule and Simpson's 1/3 rule, find the velocity of the rocket at  $t = 80$  sec. (A/M 2010)

$t(\text{sec})$ :	0	10	20	30	40	50	60	70	80
$t(\text{cm/sec})$ :	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	40.67

### • Double integrals by Trapezoidal and Simpson's rules

1. Using Trapezoidal rule, evaluate  $\int_1^2 \int_1^2 \frac{dxdy}{x^2 + y^2}$  numerically with  $h = 0.2$  along  $x$  - direction and  $k = 0.25$  along  $y$  -direction. (M/J 2012)

Textbook Page No.: 4.56

2. Evaluate  $\int_0^1 \int_0^1 \frac{1}{1+x+y} dxdy$  by trapezoidal rule. (N/D 2014)

Textbook Page No.: 4.57

3. Evaluate  $\int_1^{1.2} \int_1^{1.4} \frac{dxdy}{x+y}$  by trapezoidal formula by taking  $h = k = 0.1$ .

Textbook Page No.: 4.59

(A/M 2010),(N/D 2017)



4. Using Trapezoidal rule, evaluate  $\int_1^2 \int_1^2 \frac{dxdy}{x+y}$  with  $h = k = 0.5$ . (M/J 2016)
5. Evaluate  $\int_0^2 \int_0^1 4xy \, dxdy$  using Simpson's rule by taking  $h = \frac{1}{4}$  and  $k = \frac{1}{2}$ . (N/D 2012)
6. Evaluate  $\int_0^{1/2} \int_0^{1/2} \frac{\sin(xy)}{1+xy} dxdy$  using Simpson's rule with  $h = k = \frac{1}{4}$ .  
(M/J 2012),(M/J 2014)

Textbook Page No.: 4.64

7. Evaluate  $\int_1^{1.2} \int_2^{2.4} \frac{1}{xy} dxdy$  using Simpson's one-third rule. (M/J 2013)

Textbook Page No.: 4.64

8. Using Simpson's 1/3 rule, to evaluate  $\int_0^1 \int_0^1 \frac{dxdy}{1+xy}$  with  $\Delta x = \Delta y = 0.25$ . (A/M 2018)

## Unit – V (Numerical Solution of Ordinary Differential Eqns.)

### • Taylor's series, Euler's and Modified Euler's method

1. Using Taylor series method, find the value of  $y$  at  $x=0.1$ , if  $y$  satisfies the equation  $\frac{dy}{dx} = x^2 - y$  given that  $y=1$  when  $x=0$ , correct to 3 decimal places.

Textbook Page No.: 5.4

(N/D 2017),(N/D2018)

2. Using Taylor's series method, find  $y$  at  $x=0.1$  if  $\frac{dy}{dx} = x^2y - 1$ ,  $y(0) = 1$ .

Textbook Page No.: 5.7

3. Apply Taylor series method to find and approximate value of  $y$  when  $x = 0.1, 0.2$  given that  $\frac{dy}{dx} = x + y$ ,  $y(0) = 1$ . (M/J 2014)

4. Given  $y' = x^2 - y$ ,  $y(0) = 1$ ,  $y(0.1) = 0.9052$ ,  $y(0.2) = 0.8213$ , find  $y(0.3)$  using Taylor's series method. (N/D 2013)

5. Using Taylor method, compute  $y(0.2)$  and  $y(0.4)$  correct to 4 decimal places given  $\frac{dy}{dx} = 1 - 2xy$  and  $y(0) = 0$ , by taking  $h = 0.2$ . (A/M 2011)
6. Solve  $y' = y^2 + x$ ,  $y(0) = 1$  using Taylor series method for  $y(0.1)$  and  $y(0.2)$ .
7. By Taylor series method find  $y(0.1)$ ,  $y(0.2)$  and  $y(0.3)$  if  $\frac{dy}{dx} = x - y^2$ ,  $y(0) = 1$ . (N/D 2012)
8. Given  $\frac{dy}{dx} = 1 + y^2$ , where  $x = 0$ , find  $y(0.2)$ ,  $y(0.4)$  and  $y(0.6)$ , using Taylor series method. (A/M 2010)
9. Solve by Taylor's method to find an approximate value of  $y$  at  $x = 0.2$  for the differential equation  $\frac{dy}{dx} = 2y + 3e^x$ ,  $y(0) = 0$ . Compare the numerical solution with the exact solution. Use first three non-zero terms in the series. (N/D 2014)
- Textbook Page No.: 5.2
10. Find the value of  $y$  at  $x = 0.1$  given that  $\frac{dy}{dx} = x^2 - y$ ,  $y(0) = 1$  by modified Euler's method. (A/M 2018)
- Textbook Page No.: 5.11
11. Using modified Euler method, find  $y(0.2)$ ,  $y(0.1)$  given  $\frac{dy}{dx} = x^2 + y^2$ ,  $y(0) = 1$ . (A/M 2011)
- Textbook Page No.: 5.12
12. Use Euler's method, with  $h = 0.1$  to find the solution of  $y' = x^2 + y^2$  with  $y(0) = 0$  in  $0 \leq x \leq 5$ . (N/D 2010)
13. Solve by Euler's method, the equation  $\frac{dy}{dx} = x + y$ ,  $y(0) = 0$ , chose  $h = 0.2$  and compute  $y(0.4)$  and  $y(0.6)$ . (N/D 2013)
14. Consider the initial value problem  $\frac{dy}{dx} = y - x^2 + 1$ ,  $y(0) = 0.5$ . Compute  $y(0.2)$  by Euler's method and modified Euler method. (M/J 2012), (N/D 2014)

15. By Modified Euler's method, find  $y(0.1)$ ,  $y(0.2)$  and  $y(0.3)$  if  $\frac{dy}{dx} = x + y$ ,  $y(0) = 1$ .  
(N/D 2012)

16. Solve  $(1+x)\frac{dy}{dx} = -y^2$ ,  $y(0) = 1$  by modified Euler method by choosing  $h = 0.1$ , find  $y(0.1)$  and  $y(0.2)$ .  
(N/D 2016)

Textbook Page No.: 5.13

### • Runge – Kutta method for 1<sup>st</sup> order equations

1. Using Runge-Kutta method order four, solve  $y' = x + y$ ,  $y(0) = 1$  to find  $y(0.2)$ .

Textbook Page No.: 5.17

(A/S 2018)

2. Find  $y(0.8)$  given that  $y' = y - x^2$ ,  $y(0.6) = 1.7379$  by using R-K method of order 4, taking  $h = 0.1$ .  
(M/J 2012)

3. Using 4<sup>th</sup> order Runge-Kutta method, solve  $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ ,  $y(0) = 1$  for  $x = 0.2$  and  $x = 0.4$  with  $h = 0.2$ .  
(A/M 2010), (A/M 2011), (M/J 2013)

Textbook Page No.: 5.21

4. Use R.K Method fourth order to find the  $y(0.2)$  if  $\frac{dy}{dx} = x + y^2$ ,  $y(0) = 1$ ,  $h = 0.1$ .  
(N/D 2010), (N/D 2017)

5. Solve  $\frac{dy}{dx} = xy + y^2$ ,  $y(1) = 1$ , for  $y(1.1)$ , using fourth order Runge-Kutta method.  
(N/D 2014)

Textbook Page No.: 5.19

### • Milne's predictor – corrector methods for 1<sup>st</sup> order eqn.

1. Given  $\frac{dy}{dx} = x^3 + y$ ,  $y(0) = 2$ ,  $y(0.2) = 2.073$ ,  $y(0.4) = 2.452$ ,  $y(0.6) = 3.023$ , compute  $y(0.8)$  by Milne's method.  
(N/D 2017)

Textbook Page No.: 5.31

2. If  $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ ,  $y(0) = 1$ , find  $y(0.2)$ ,  $y(0.4)$  and  $y(0.6)$  by Runge-Kutta method of fourth order and hence find  $y(0.8)$  by Milne's method. (N/D 2012), (M/J 2016)

3. Compute  $y(0.4)$  and  $y(0.5)$ , given that  $y' = y - \frac{2x}{y}$ ,  $y(0) = 1$ ,  $y(0.1) = 1.0954$ ,  $y(0.2) = 1.1832$ ,  $y(0.3) = 1.2649$  using Milne's predictor-corrector method. (N/D 2014)

4. If  $\frac{dy}{dx} = x^2 + y^2$ ,  $y(0) = 1$  find  $y(0.1)$ ,  $y(0.2)$  and  $y(0.3)$  by Taylor series method. Hence find  $y(0.4)$  by Milne's Predictor-Corrector method. (M/J 2016)

5. Using Milne's predictor and corrector method find  $y(4.4)$  given  $5xy' + y^2 - 2 = 0$  given  $y(4) = 1$ ,  $y(4.1) = 1.0049$ ,  $y(4.2) = 1.0097$ ,  $y(4.3) = 1.0143$ . (M/J 2012),(M/J 2014),(A/M 2018)

Textbook Page No.: 5.28

6. Using Milne's method, obtain the solution of  $\frac{dy}{dx} = x - y^2$  at  $x = 0.8$  given  $y(0) = 0$ ,  $y(0.2) = 0.02$ ,  $y(0.4) = 0.0795$ ,  $y(0.6) = 0.1762$ . (N/D 2010)

Textbook Page No.: 5.25

7. Using Milne's predictor-corrector method, find  $y(0.4)$ , given that  $y' = \frac{(1+x^2)y^2}{2}$ ,  $y(0) = 1$ ,  $y(0.1) = 1.06$ ,  $y(0.2) = 1.12$ ,  $y(0.3) = 1.21$ . (N/D 2013)

Textbook Page No.: 5.26

## Book for Reference:

“Statistics and Numerical Methods”

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